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Essays in nonparametric measures of changes in taste and hedging behavior with options

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Iowa State University, 1991
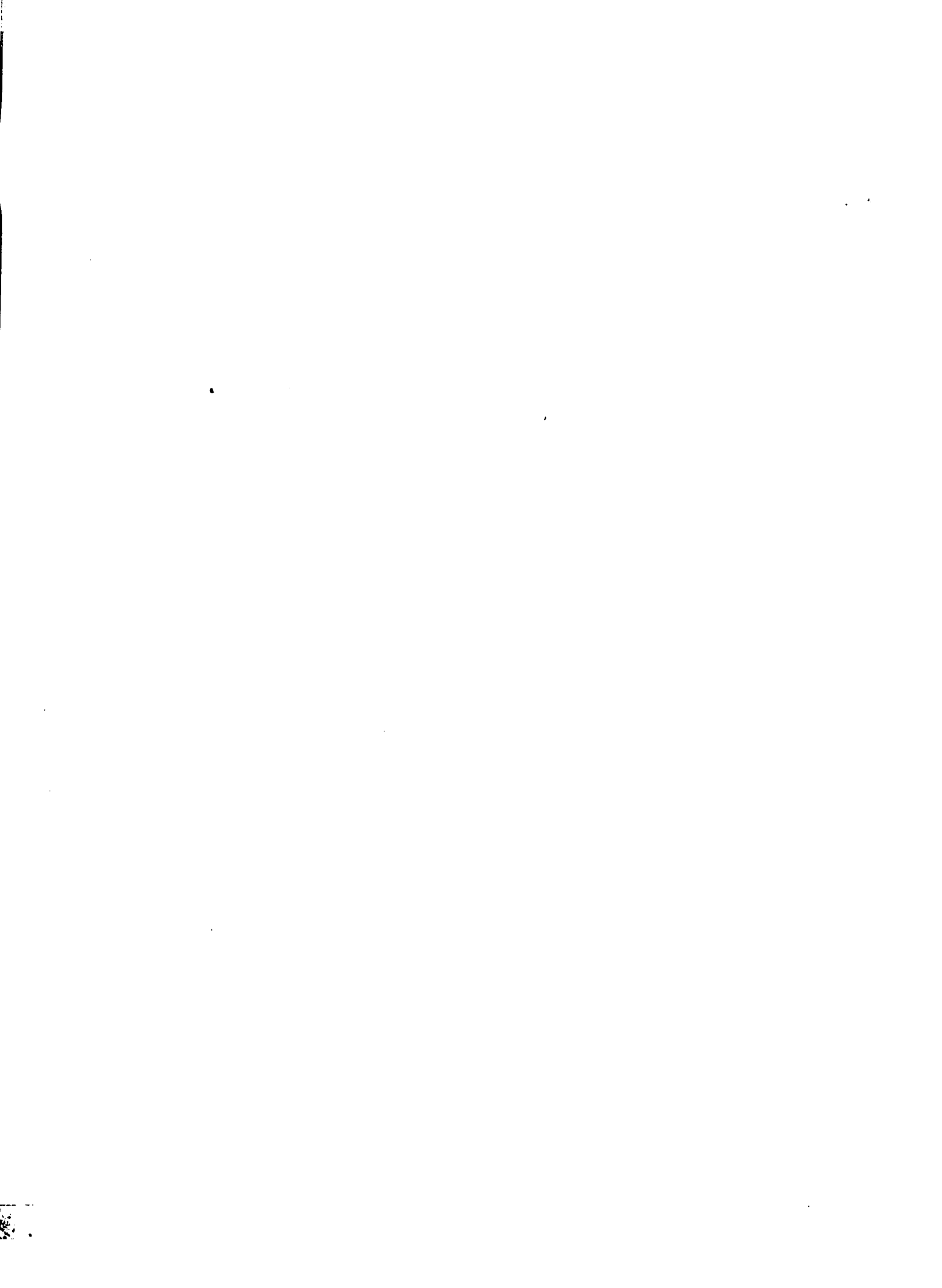
Essays in nonparametric measures of changes in taste

and hedging behavior with options

by

Yong Sakong

A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Approved:

In Charge of Major Work

For the Major Department

For the Graduate College

Iowa State University
Ames, Iowa

1991

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GENERAL INTRODUCTION

This dissertation consists of three independent papers. The first paper presents a way of detecting violation of consumer preference theory that has some of the advantages of existing parametric and nonparametric methods. The proposed method does not require any subjective input on behalf of the modeler and is therefore less subject to pretesting and data mining. The new method has the ability to detect slight violations in preferences, even when the budget constraint has shifted out, a feature that has not been found in the nonparametric models that have been presented to date.

The second paper examines behavior of an expected utility maximizing individual who faces both price and production uncertainty and who has access to both futures and options markets. The key insight is that the mean value theorem can be used to solve expected utility maximization problems when the price distribution is truncated. The results show that firms will almost always use options and that the firm will hedge more or less in the futures market than it would in the absence of production uncertainty. The results also show that mean variance analysis produces a good result so long as markets are perceived to be unbiased and if there is no production uncertainty. The error caused by the improper use of mean variance analysis when production uncertainty exists can be quite large.

The third paper shows that options have a role to play as a hedging instrument when production uncertainty is introduced. Options are useful whether or not producers believe that their individual yields are correlated with market prices. In addition, the usefulness of options as hedging tools increase with firm-specific production uncertainty and for producers who are more risk averse at lower revenues.

Explanation of Dissertation Format

This dissertation follows the ISU format for the alternate style. Because each paper was written for, and submitted to paper review journals, each contains an introduction and conclusion as well as a review of the relevant literature. An overall conclusion is included following the third paper. The first and third papers follow the *American Journal of Agricultural Economics* format and the second paper follows the *Economics Letters* format.

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PART I. A TEST FOR THE CONSISTENCY OF DEMAND DATA
WITH CONSUMER PREFERENCE THEORY

INTRODUCTION

Recent papers by Alston and Chalfant (1991a, 1991b, 1991c), Chalfant and Alston, and Cox and Chavas (1987, 1990) have cast doubt on the parametric techniques previously used for measuring taste changes. Alston and Chalfant (1991c) most graphically demonstrated the problem with parametric methods by showing that one could detect structural change in beef demand almost 100 percent of the time in a system in which, by construction, no such change exists. In their paper, data were generated by a linear demand system, and the structural change test was conducted with a logarithmic and Almost Ideal Demand System (AIDS) model.

This issue is very similar to that described by Leamer. Taste change effects, if they exist, will be relatively small. To measure these changes by using parametric methods, the researcher is forced to make decisions about the functional form and the estimation procedure. If the researcher is looking for (or if the system rewards) evidence of taste change, he or she need only search among the set of inferences one can draw from a particular data set for those results that are most pleasing.

The proposed solution in the nonparametric literature is to avoid to the greatest possible extent all decisions that might influence the possible outcome. For demand analysis, this solution simply involves testing the data for consistency with the weak or strong axioms of revealed preference. These axioms avoid the need to express and estimate the direct or indirect expenditure functions and instead rely on very intuitive conditions. These conditions state that, in the absence of taste change, if a bundle of Q_1 is revealed preferred to a bundle of Q_2 at one
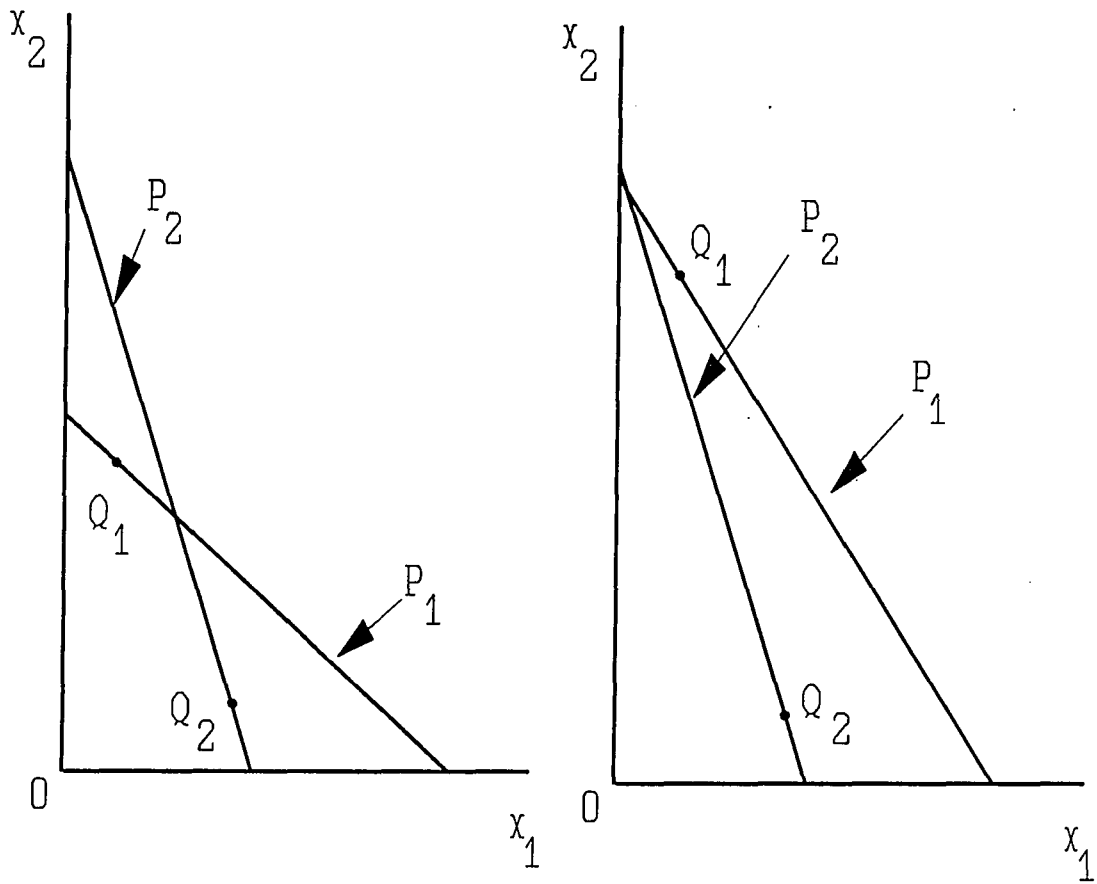
Figure 1. A two-good example of Weak Axiom of Revealed Preference (WARP)

point, then Q_2 cannot be revealed preferred to Q_1 at another point, unless taste change has occurred.[1]

One problem that occurs with real data is that, as real income has increased, real expenditures on some commodity bundles have also increased. This trend makes it very difficult to find a bundle that was affordable and not consumed in one period but that was consumed later although the previous bundle remained affordable. This can best be demonstrated in Figure 1, where the left-hand panel shows a violation of consumer preference theory and the right-hand panel shows a similar situation but with the budget constraint shifted out. Suppose that the initial consumption bundle and price vector are Q_1 and P_1 and that the consumer chooses bundle Q_2 when the price vector changes from P_1 to P_2. Intuitively, tastes have shifted away from good x_2 to good x_1 in both situations. For a violation of consumer preference, the budget lines must cross. Hence, the situation in the right-hand panel does not provide evidence of a violation of consumer preference theory.[2] The movement away from x_2 could be explained by an almost vertical Engel curve, and the movement toward x_1 might have occurred because x_1 is an inferior good.

Consider Figure 2. Here, as discussed earlier, the movement from Q_1 to Q_2 can be explained by assuming that x_1 is an inferior good. Suppose that we are prepared to assume that x_1 is not an inferior good, then the question arises as to whether the consumption change is

---

[1] Consider a batch of commodities, Q_1, purchased at prices P_1. Now consider a second commodity bundle, Q_2, such that $P_1Q_2 \leq P_1Q_1$. Because the consumer could afford Q_2 at prices P_1 but chose Q_1 instead, Q_1 is revealed preferred to Q_2. The weak axiom states that, at prices P_2, we will not see $P_2Q_1 \leq P_2Q_2$, i.e., Q_2 will not be revealed preferred to Q_1 at any set of prices. The strong axiom introduces transitivity, i.e., if Q_1 is revealed preferred to Q_2 and Q_2 is revealed preferred to Q_3, then Q_3 must never be revealed preferred to Q_1.

[2] Alston and Chalfant (1991a) recently showed that the probabilities of violating the Weak Axiom of Revealed Preference (WARP) tend to increase as the size of the taste change increases and as the growth rate of total expenditures decreases.
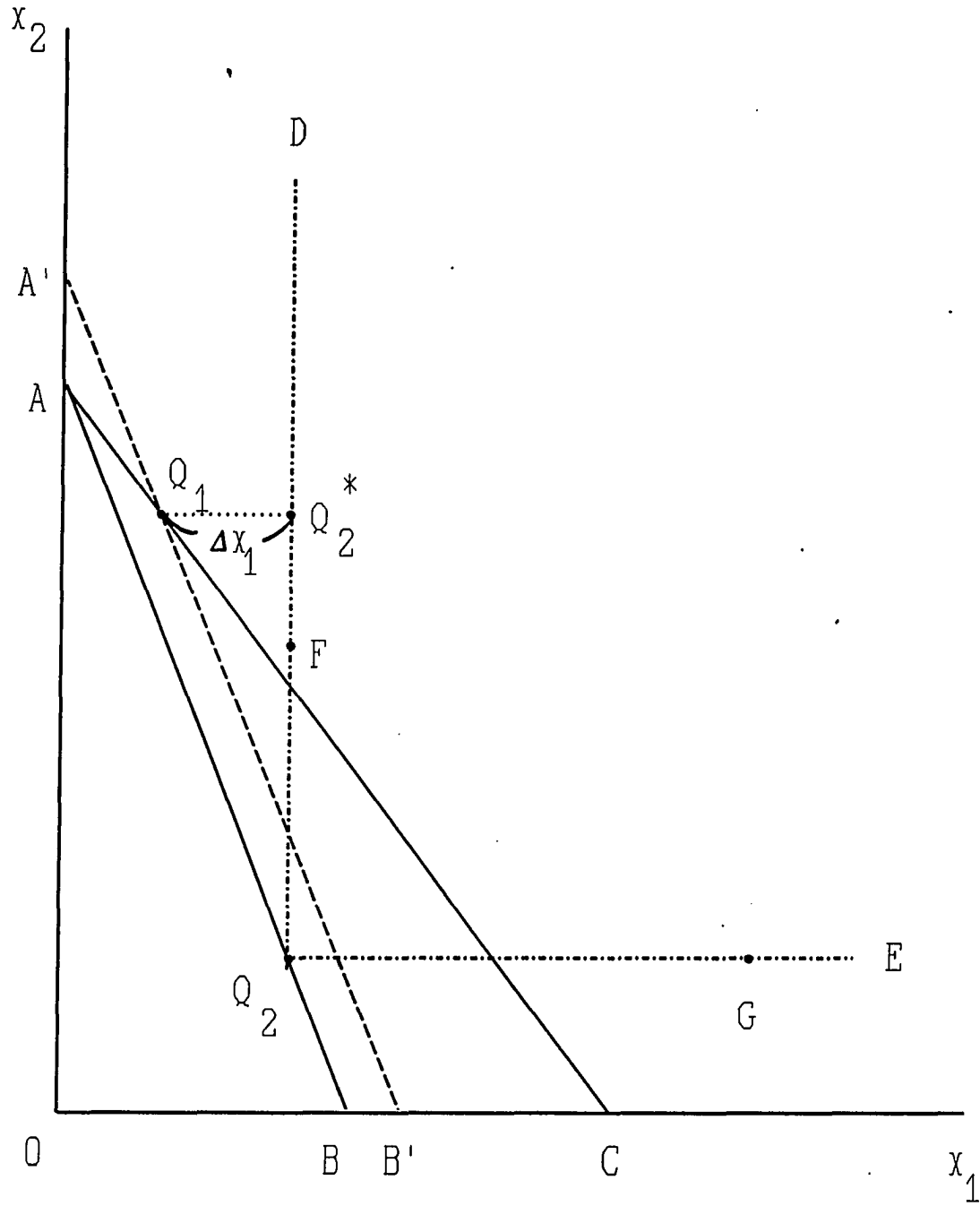
Figure 2. An example of a violation of consumer preferences

consistent with consumer preference theory. If consumption patterns are not consistent, then we may conclude that tastes (or consumer preferences) have changed. One very obvious implication of consistent preferences is that if consumers are held on the same indifference curve and subjected to the same price vector, then they should consume the same bundle. However, we can show that so long as x_1 is not inferior, the range of possible bundles found by adjusting for the price effect from Q_1 does not overlap with the range found when the expenditure effect is compensated from Q_2.

To show that two regions cannot overlap, consider how actual demand would change if we start at Q_1 and change the prices to P_2 and then compensate the consumer for the price change. Because we do not know the shape of the indifference curve, we draw the new budget line A'B' to allow for the maximum possible compensation; that is, we allow the consumer to purchase Q_1 at the new price line. In reality, the new budget line will lie to the left of A'B' and the new compensated bundle, $h(U_1, P_2)$, will lie in the region AQ_1A'. Now if we start at Q_2 and compensate for the price increase, the new bundle, $g(U_1, P_2)$, will lie in the region DQ_2E. This is true because the consumer will spend the additional compensatory income on both non-inferior goods. We have created a situation with two demands, $h(U_1, P_2)$ and $g(U_1, P_2)$. If consumer preferences are consistent, then the two solutions should be identical. Yet as we have shown graphically, the two regions do not even overlap.

To measure this change in tastes, we ask which set of expenditure elasticities best explains the behavior, the remainder being attributed to taste change. This gives us the minimum taste change that explains the data. In this very simple example, one might conclude that the income elasticities are such that all the additional compensatory income is spent on x_2; this would lead to Q_2^*. The minimum taste change is therefore away from x_1 by the horizontal amount $Q_1 - Q_2^*$

measured in units of x_1 as shown by Δx_1 in Figure 2. This example motivates the test that

follows. We minimize the degree of taste change that satisfies both consistency of preferences and

the restrictions we place on these expenditure elasticities. This procedure is a little more

complicated than that indicated by Figure 2. This is because the test minimizes the amount of taste

change required to satisfy convexity of preferences, non-negativity and adding-up, while the

intuition developed in Figure 2 uses only the convexity and non-negativity. However, the

intuition remains the same.

In the analysis that follows we progressively impose restrictions on the slopes of the Engel

curves for meat data from the United States, Canada, Japan, and South Korea. First we impose

non-negativity, adding-up, and convexity. Then, reasonable ranges for the expenditure elasticities

are imposed. Finally, we impose restrictions on how the expenditure elasticities can change from

year to year. In all cases, we simultaneously estimate the minimum consumption changes needed

to satisfy consistency of preferences and the expenditure elasticities that best explain this behavior.

The mechanism we use to measure the degree to which preference consistency is violated is based

on a linear programming model recently developed by Cox and Chavas (1987, 1990) and Chavas

and Cox. We introduce into their model the modifications needed to simultaneously solve for the

set of expenditure elasticities that minimizes taste change and the amount of taste change itself as

well as to impose restrictions on expenditure elasticities. In the empirical analysis, we use the new

method, which we call a test for consistency of preferences, to detect and measure taste change in

the countries mentioned above.

A TEST FOR CONSISTENCY OF PREFERENCES

Suppose that n goods exist and that demand for good i, x_i, is a function of prices, income, and taste as follows:

$$x_i = f_i(P', y, T') \tag{1}$$

where P and T are price and taste vectors, respectively; that is, $P' = (p_1, p_2, \cdots, p_n)$ and $T' = (t_1, t_2, \cdots, t_n)$, and y is income (or expenditure). If we differentiate equation (1), we find:

$$\Delta x_i = \sum_{j=1}^{n} \frac{\partial f_i}{\partial p_j}\Delta p_j + \frac{\partial f_i}{\partial y}\Delta y + \sum_{j=1}^{n} \frac{\partial f_i}{\partial t_j}\Delta t_j. \tag{2}$$

Using the Slutsky equation and temporarily assuming that $\Delta t_j = 0$ for all $j = 1, 2, \cdots,$ n, equation (2) can be rewritten as:

$$\Delta x_i = \sum_{j=1}^{n} \frac{\partial f_i}{\partial p_j}\Big|_{U_0}\Delta p_j + a\frac{x_i}{y}\epsilon_{iy} \tag{3}$$

where $a \equiv \Delta y - \sum_{j=1}^{n} x_j\Delta p_j$ and ϵ_{iy} is an expenditure elasticity of good i.

Equation (3) separates the demand change induced by price changes and expenditure change into two effects: the first part is the substitution effect induced by price changes and the second is the expenditure effect induced by both price changes and expenditure changes. Subtracting the second part in the right-hand side of equation (3) from the observed demand data, $x_i^* = x_i - a(x_i/y)\epsilon_{iy}$, x_i^* is a compensated demand for good i. By holding the consumer on the same indifference curve in this manner, we can respecify the conditions under which consistency
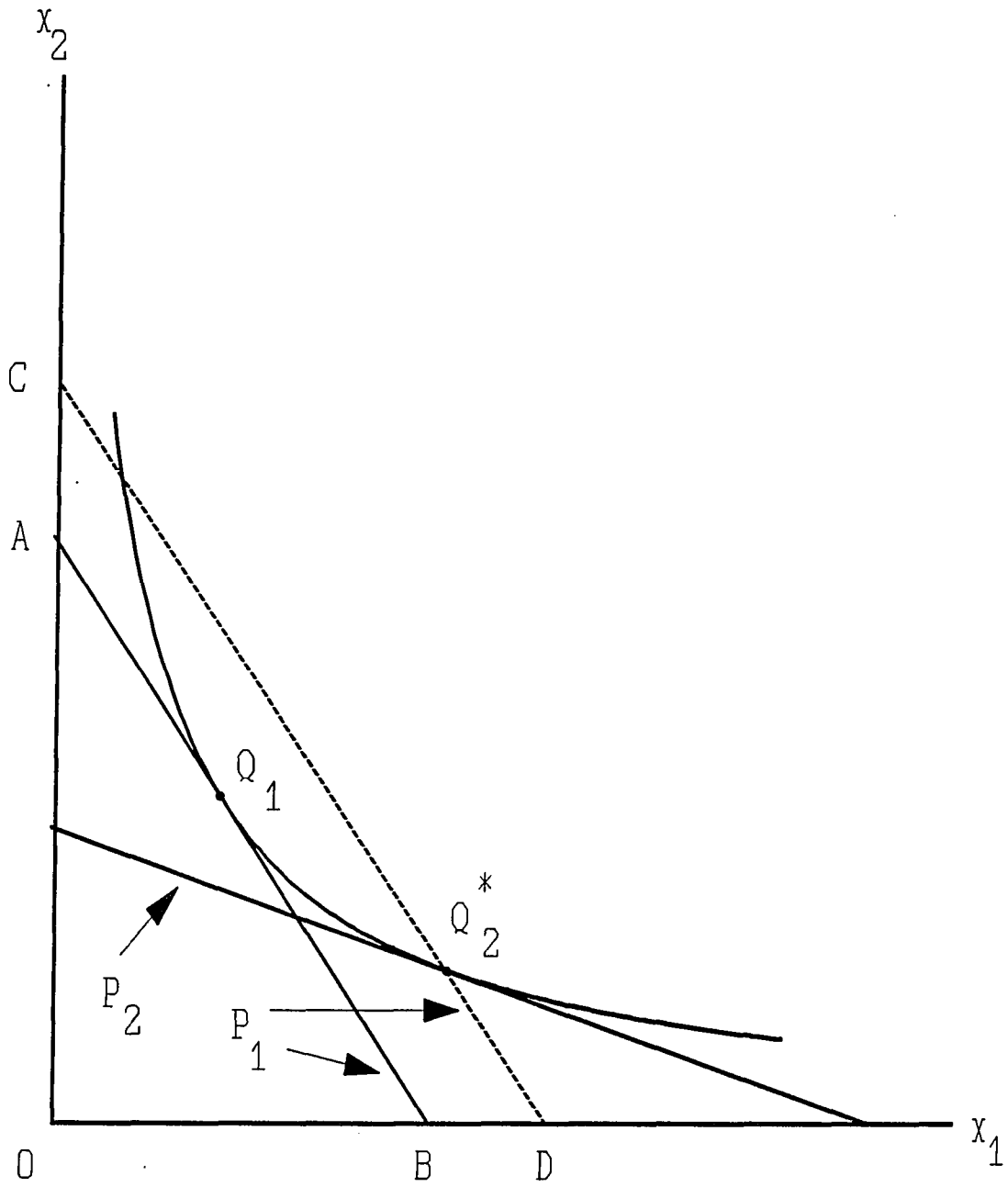
Figure 3. Graphical representation of the convexity condition

of preferences is violated as $P_t'Q_t^* > P_t'Q_s^*$ for all t and s, where Q_t^* is a compensated

consumption bundle at time t, i.e., $Q_t^{*\prime} = (x_{1t}^*, \ldots, x_{nt}^*)$.[3] To see why this is true, consider

Figure 3. Here we consider two consumption bundles, Q_1 and Q_2^*. Bundle Q_1 is a base-year

consumption bundle and Q_2^* is the optimal consumption bundle at time 2 prices and the time 1

utility level. Note that if the indifference curve is convex, then Q_2^* will lie to the right of AB.

Now if we measure the expenditures of Q_2^* and Q_1 in time 1 prices, then expenditure $P_1'Q_2^*$

will be equal to or greater than $P_1'Q_1$. To see why this is true, draw a line through Q_2^* parallel

to AB (CD in Figure 3) and measure expenditure in terms of good 2. If $P_1'Q_2^*$ is less than or

equal to $P_1'Q_1$ (i.e., OC < OA), then preferences are inconsistent with the data. Because the

inequality $P_1'Q_1 \leq P_1'Q_2^*$ depends on the convexity of the indifference curve, we call this the

convexity condition.[4]

Suppose that there are two goods, A and B, and that a positive taste change occurs in one

good and a negative taste change occurs in the other good. When two goods are assumed to be

substitutes, then a positive taste change in one good will decrease demand for the other good.

Unfortunately, we cannot distinguish whether a taste change in good A causes demand for good B

to change or whether a taste change in good B causes demand for good A to change. Therefore,

taste change in good i in our model can be explained by $\sum_{j=1}^{n} (\partial f_i/\partial t_j)\Delta t_j$ rather than only by

$(\partial f_i/\partial t_i)\Delta t_i$. That is, the taste change in terms of good i is measured by the changes in the demand

for the good, which cannot be interpreted by the substitution and expenditure effects, even though

it may have been caused by taste changes in other goods. Therefore, the third term of the right-

---

[3] If t or s is the base year, Q^* is a consumption bundle rather than a compensated consumption
bundle.

[4] Varian shows a similar condition for the cost-minimizing firm.

hand side in equation (2) will be simply expressed as tc_j; $tc_j = \sum_{j=1}^{n} (\partial f_i/\partial t_j)\Delta t_j$.

Some values of TC always exist to satisfy the inequality $P_t'(Q_t^* - TC_t) \leq P_t'(Q_s^* - TC_s)$

for all t and s, where $TC_t' = (tc_{1t}, tc_{2t}, \cdots, tc_{nt})$.[5] We can find the minimum TC that satisfies

these inequalities by solving the following problem:[6]

$$\underset{TC, \phi}{Min} \quad b' \, TC \qquad\qquad (4)$$

$$s.t. \quad P_t'(Q_t^*-TC_t) \leq P_t'(Q_s^*-TC_s) \quad \textit{for all t and s}$$

$$\sum_i w_i^t \epsilon_{iy}^t = 1 \quad \textit{for all t}$$

$$\epsilon_{iy}^t \geq 0 \quad \textit{for all i and t}$$

where ϕ is a vector of expenditure elasticities, b is arbitrarily defined such that problem (4) is

bounded, and w_i^t and ϵ_{iy}^t are an expenditure share and an expenditure elasticity of good i at time

t. The first and second constraints represent the convexity and adding-up condition, respectively.

The third constraint represents the non-negativity of expenditure elasticities.

To see how the adding-up condition influences the results, consider that, in the simple

two-good model, the compensated demands of goods 1 and 2 at time 2, x_1^* and x_2^*, are:

---

[5] Taste change may be negative or positive so that $(TC^+ - TC^-)$ is actually substituted into TC in the linear programming problem, where $TC^+ \geq 0$ and $TC^- \geq 0$.

[6] Chavas and Cox test for technical changes by using a similar method.

$$x_1^* = x_1 - \frac{a}{y}x_1\epsilon_{1y} \tag{5}$$

$$x_2^* = x_2 - \frac{a}{y}x_2\epsilon_{2y} \tag{6}$$

and the adding-up condition is:

$$w_1\epsilon_{1y} + w_2\epsilon_{2y} = 1 \tag{7}$$

Because equations (5) through (7) have four unknown variables and three equations, we can obtain the following relationship:

$$x_2^* = \Pi_1 - \Pi_2 x_1^* \tag{8}$$

$$\textit{where} \quad \Pi_1 = x_2(1 - \frac{a}{y}\frac{x_2}{w_2} + \frac{w_1}{w_2})$$

$$\Pi_2 = \frac{w_1}{w_2}\frac{x_2}{x_1}$$

Because Π_1 and Π_2 are known coefficients and Π_2 is positive, the compensated demand of good 2, x_2^*, has a linear negative relationship with x_1^*. Now suppose in Figure 2 that this relationship was satisfied along the line that connects Q_2^* and G. Then, the restriction has no impact as it allows the vertical move from Q_2 to Q_2^*. Suppose, however, that the expenditure elasticities that underlie the move from Q_2 to Q_2^* violated adding-up; this is equivalent to stating that the relationship between x_2^* and x_1^* intersect at some point other than at Q_2^*, say at point F. In this

case, the minimum taste change would be Q_1 - Q_2^* units of x_1 and Q_2^* - F units of x_2.

One practical problem remains: because Δ represents a very small change, Δy, Δx, and Δp in equations (2) through (8) must also be small changes. To address this, we set the year of first observations as a base year, or time 1. We then denote the partial expenditure effect of good i at time t (pee_{it}) as an expenditure effect that occurs when consumption bundles and their prices at time t are compared with those at a previous time (t-1). Then the expenditure effect on good i at time t, (ee_{it}), is the sum of partial expenditure effects from time 2 to time t:

$$ee_{it} = \sum_{j=2}^{t} pee_{ij} = \sum_{j=2}^{t} a_j \frac{x_{ij}}{y_j} \epsilon_{iy}^j \tag{9}$$

Similarly, we let the partial taste change of good i at time t (ptc_{it}) be a taste change that occurred between (t-1) and t. Taste change of good i at time t (tc_{it}) is therefore also measured as the sum of the partial changes from time 2 to time t:

$$tc_{it} = \sum_{j=2}^{t} ptc_{ij} \tag{10}$$

In effect, the taste change at time t would be the accumulation of past taste changes as well as a current taste change when the current consumption bundle is compared with that of the base year.

Substituting (9) into (3) and rearranging, the compensated demand for good i at time t

is:[7]

$$x_{it}^* = x_{it} - \sum_{j=2}^{t} a_j \frac{x_{ij}}{y_j} \epsilon_{iy}^j \tag{11}$$

By substituting (10) and (11) into (4), we avoid the need for estimating x_{it}^*. The model we actually solve is:

$$\underset{PTC, \phi}{Min} \quad b' \, PTC \tag{A}$$

$$s.t. \quad (i) \quad \sum_{i=1}^{n} p_{it} x_{it} - \sum_{i=1}^{n} p_{it} x_{is}$$

$$\leq \sum_{i=1}^{n} \sum_{j=2}^{t} a_j \frac{p_{it} x_{ij}}{y_j} \epsilon_{iy}^j - \sum_{i=1}^{n} \sum_{j=2}^{s} a_j \frac{p_{it} x_{ij}}{y_j} \epsilon_{iy}^j$$

$$+ \sum_{i=1}^{n} \sum_{j=2}^{t} p_{it} ptc_{ij} - \sum_{i=1}^{n} \sum_{j=2}^{s} p_{it} ptc_{ij}$$

for all t and s

$$(ii) \quad \sum_i w_i^t \epsilon_{iy}^t = 1 \qquad \text{*for all t*}$$

$$(iii) \quad \epsilon_{iy}^t \geq 0 \quad \cdot \text{*for all i and t*}$$

where PTC is a vector of partial taste changes; $PTC' = (ptc_{11}, ptc_{21}, \cdots, ptc_{n1}, \cdots, ptc_{1T}, ptc_{2T}, \cdots, ptc_{nT})$.

---

[7] x_{it}^* in (11) is a compensated demand for good i if there is no taste change. This condition was relaxed by introducing taste change terms into (4). Another way of introducing taste change is to subtract $\sum_{j=2}^{t} ptc_{ij}$ from the right-hand side of (11). The results in both cases are the same.

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EMPIRICAL APPLICATION

Data

Data on per capita annual consumption of beef, pork, and chicken during 1971 through 1984 for four countries (the United States, Canada, Japan, and South Korea) are used. The U.S. data is taken from Chalfant, the Canadian data from Van Kooten, and the Japanese data from Wahl and Hayes. The South Korean data is collected from the annual reports of the Agricultural Cooperative Federation and the National Livestock Cooperatives Federation. Because of the enormous number of restrictions necessary to solve the model, we were limited to 15 years of data. We choose the 15 years that all four data sets had in common. This centers the U.S. data around the 1976 to 1978 period and therefore includes the years of maximum U.S. beef consumption as well as the decrease in consumption that triggered the series of taste change studies mentioned earlier.

Results and Discussion

The results obtained from U.S. meat demand data using the model (A) are presented in Figure 4. This figure represents the per capita change in pounds from the base year of 1971. One of the more interesting features of these results is the gradual trend away from beef. As the program is written, each year is treated independently; therefore, years in which taste moved in favor of beef can in practice be followed by years in which the movement was against beef. The existence of a trend away from beef would seem to indicate that the source of the inconsistency—be it data driven, or caused by health concerns—is not random.

The beef results indicate a cumulative movement away from beef of approximately 3.5 lb per capita with most (2 lb) occurring from 1972 to 1973. Actual per capita U.S. beef
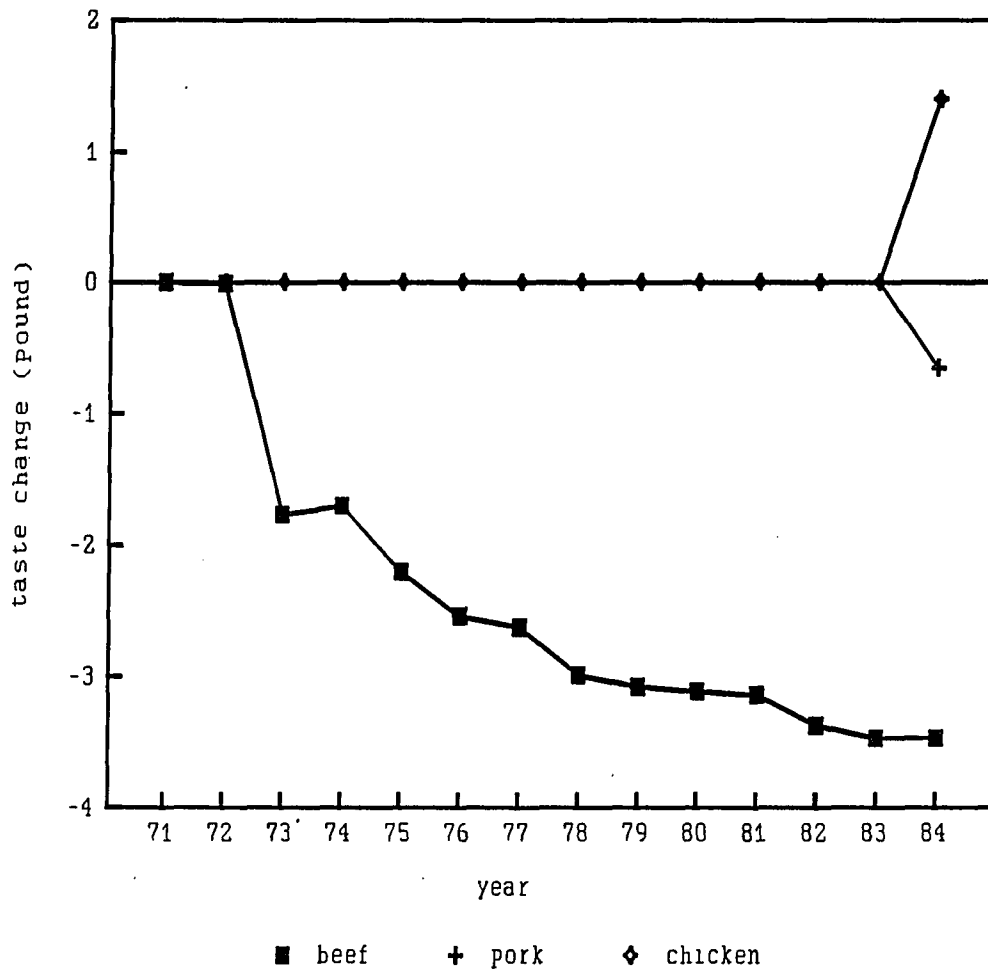
Figure 4. Taste change in U.S. meat demand

Table 1. Expenditure elasticities required to minimize the taste changes in U.S. meat demand

Year	Beef	Pork	Chicken
1972	0.00	0.00	9.62
1973	1.78	0.02	0.00
1974	1.76	0.00	0.00
1975	1.17	0.00	2.85
1976	0.12	1.54	4.52
1977	0.00	0.00	8.40
1978	0.00	0.00	8.62
1979	0.00	0.00	8.85
1980	1.06	0.00	3.33
1981	0.00	3.22	0.00
1982	0.00	0.21	8.06
1983	0.00	0.00	8.47
1984	0.00	0.00	7.35

consumption from 1971 to 1980 was 83.4, 85.4, 80.5, 85.6, 87.9, 94.4, 91.8, 87.2, 78.0, and

76.5 lb, respectively. The results for U.S. pork and chicken consumption are consistent with

theory, with small violations against pork and toward chicken in 1984.

In the above model, we imposed only adding-up and non-negativity restrictions on

expenditure elasticities. The expenditure elasticities that underlie Figure 4 are shown in Table 1.

As mentioned, these elasticities are found by minimizing the amount of taste change. The program

makes no attempt to realistically measure these elasticities other than to ensure that they satisfy

adding-up and non-negativity. The expenditure elasticities of chicken seem unreasonably high.

This motivates the imposition of restrictions on the expenditure elasticities discussed next.

If we knew the true expenditure elasticities, our test results would be more accurate than

the results obtained from using (A). If we attempt to measure these elasticities, however, the

model misspecification problem will be reintroduced. To minimize this disadvantage, we now

introduce statistical confidence intervals of estimated expenditure elasticities. The hope in doing so

is that errors arising from model misspecification can be minimized. The model (A) can be

rewritten when the upper and lower bounds of expenditure elasticities are considered:

$$\underset{PTC,\phi}{Min} \quad b' \ PTC \qquad\qquad (B)$$

$$s.t. \quad (i) \ \sum_{i=1}^{n} p_{it}x_{it} - \sum_{i=1}^{n} p_{it}x_{is}$$

$$\leq \sum_{i=1}^{n} \sum_{j=2}^{t} a_j \frac{p_{it}x_{ij}}{y_j}\epsilon_{iy} - \sum_{i=1}^{n} \sum_{j=2}^{s} a_j \frac{p_{it}x_{ij}}{y_j}\epsilon_{iy}$$

$$+ \sum_{i=1}^{n} \sum_{j=2}^{t} p_{it}ptc_{ij} - \sum_{i=1}^{n} \sum_{j=2}^{s} p_{it}ptc_{ij}$$

for all t and s

$$(ii) \sum_{i} w_i^t \epsilon_{iy}^t = 1 \qquad \text{for all } t$$

$$(iii) \ \epsilon_{iy}^t \geq 0 \quad \text{for all } i \text{ and } t$$

$$(iv) \ \mu^L \leq \phi \leq \mu^U$$

where μ^L and μ^U are vectors of lower bounds and upper bounds, respectively, of estimated expenditure elasticities. The expenditure elasticity of good i, which is derived from (5) or (6), has lower and upper bounds ϵ_{iy}^L and ϵ_{iy}^U:

$$\epsilon_{iy}^L \leq \epsilon_{iy} = \frac{(x_i - x_i^*)y}{ax_i} \leq \epsilon_{iy}^U$$

and thus the compensated demand of good i has a narrower range than does (A), as follows:

$$x_i - \frac{ax_i}{y}\epsilon_{iy}^U \leq x_i^* \leq x_i - \frac{ax_i}{y}\epsilon_{iy}^L \quad \text{for all } i$$

The expenditure elasticities of meat demand are estimated from the AIDS model of Deaton and Muellbaur as:

$$w_i^t = \alpha + \sum_{j=1}^n \beta_{ij}\log(p_{jt}) + \beta_i\log(\frac{y}{P^*}) + e_{it}$$

where P^* is a price index approximated by the Stone geometric index; that is, $\log(P^*) = \sum_{i=1}^n w_i\log(p_i)$, and e_{it} is an error term. The time period estimating the expenditure elasticities is 1960-85.

Table 2. Upper and lower bounds of expenditure elasticities at the means

	United States		Canada		Japan		South Korea	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Beef	1.042	1.369	1.149	1.452	0.595	1.780	0.351	0.783
Dairy					1.206	1.789		
Pork	0.462	0.906	0.442	0.860	0.316	0.692	1.394	1.869
Chicken	0.392	1.374	0.529	1.022	1.258	1.543	-0.382	0.760
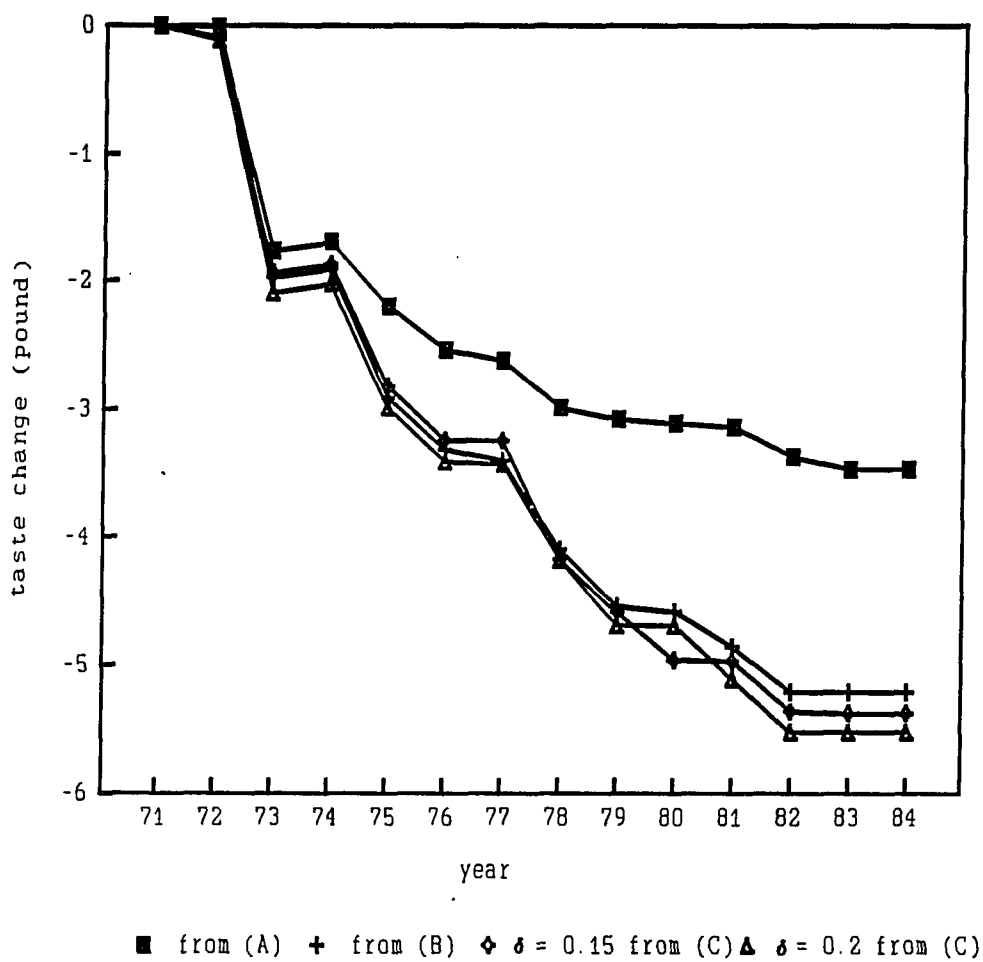
Figure 5. Taste change in U.S. beef demand

Table 2 shows the upper and lower bounds of expenditure elasticities at the mean when confidence intervals of 95 percent are used.[8] The U.S. expenditure elasticities for beef range from 1.042 to 1.369, and the chicken elasticities range from 0.392 to 1.374. These elasticities seem more reasonable than those presented in Table 1.

The results obtained from U.S. meat data using model (B) are shown in Figure 5, where only the taste changes of beef are represented for graphical convenience. These results indicate a movement of slightly more than 5 lb against beef. The unreported results for chicken show a positive movement of 2.6 lb between 1979 and 1984. These numbers are not dramatically different from the results of the first test despite the very restrictive impact of this procedure on the magnitude of the chicken expenditures.

A second way of imposing realism on the elasticities from Table 1 is to impose reasonable bounds on how elasticities can change from year to year. For example, Table 1 indicates that the expenditure elasticity for chicken was 9.62 in 1972 and 0.00 in 1973. This result motivates a restriction on the magnitude of the year-to-year changes in expenditure elasticities. This procedure does not depend on any parametric estimates. Suppose that we impose the restriction that the difference of the expenditure elasticities between time t and the previous time (t-1) for all t is less than $\pm\delta$. Then, $|\epsilon_{iy}^{t} - \epsilon_{iy}^{t-1}| \leq \delta$ is used in place of the fourth constraint in (B) to get model (C).

The smaller δ is, the larger the magnitude of taste changes. In our applications, $\delta = 0.15$ and 0.2 for all i and t, respectively. That is, the changes of expenditure elasticities at time t are allowed to change from $\epsilon_{iy}^{t-1} - 0.15$ to $\epsilon_{iy}^{t-1} + 0.15$ and from $\epsilon_{iy}^{t-1} - 0.2$ to $\epsilon_{iy}^{t-1} + 0.2$,

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[8] We actually used different lower and upper bounds of expenditure elasticities every year because $\epsilon_{iy}^{t} = 1 + \beta_{i}/w_{i}^{t}$ and w_{i}^{t}'s are different every year.

respectively.

The results obtained from model (C) for U.S. beef data are also shown in Figure 5. The lines " $\Diamond$ - $\Diamond$ - $\Diamond$ " and " Δ - Δ - Δ " show the taste changes obtained by using $\delta = 0.15$ and $\delta = 0.2$ in problem (C). The taste changes in both cases are almost identical.

One could also place the year-to-year restriction on the estimated elasticities of the second procedure; however, this procedure does not change the results in any significant way.

For the United States, one may conclude that some consistent bias has existed against beef. The cumulative effect of this bias has been somewhere between 3.5 and 5.0 lb over the period of the study. Interestingly this is similar to the result found by Moschini and Meilke when using parametric techniques. We cannot tell if this inconsistency is attributable to some systematic error in the data, for example, a gradual underreporting of the amount of fat cut off beef, or because consumer preferences have in fact moved against beef. The magnitude of this bias seems small, however, when compared to the more than 15-lb decrease in consumption observed between 1976 and 1984.

The U.S. results demonstrate the ability of the new method to detect relatively small changes in preferences. Given the standard errors usual in parametric work, it is unlikely that one could ever provide convincing evidence of a one- or two-pound per capita change in preferences. Also, neither Chalfant and Alston nor Cox and Chavas detected any taste change when nonparametric methods were used.

Figures 6 through 8 show results from (A) and (B) for Canada, Japan, and South Korea, respectively. These results are expressed in kilograms per capita. The results for Canada are very similar to those for the United States, with a maximum shift against beef of 3 kg and a move in favor of poultry of almost 3 kg. The Canadian results indicate a slight movement away from pork
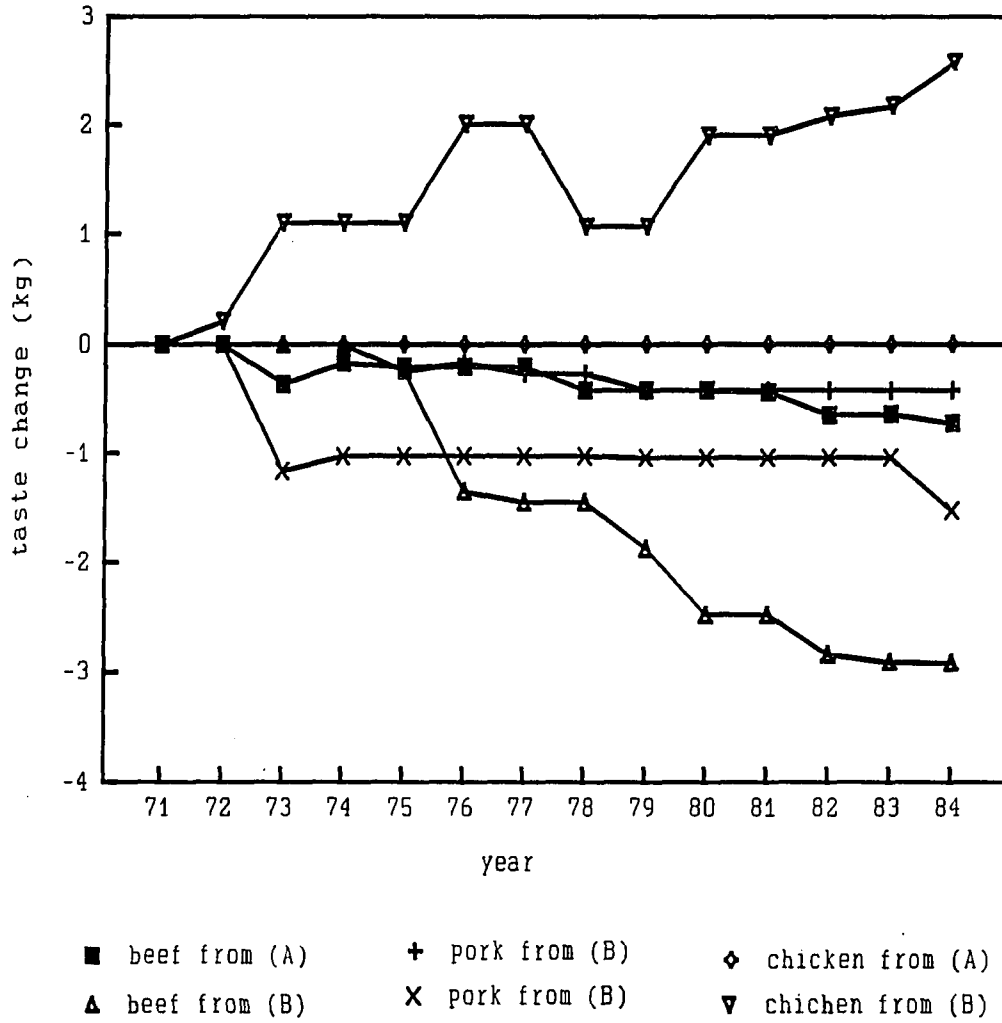
Figure 6. Taste change in Canadian meat demand
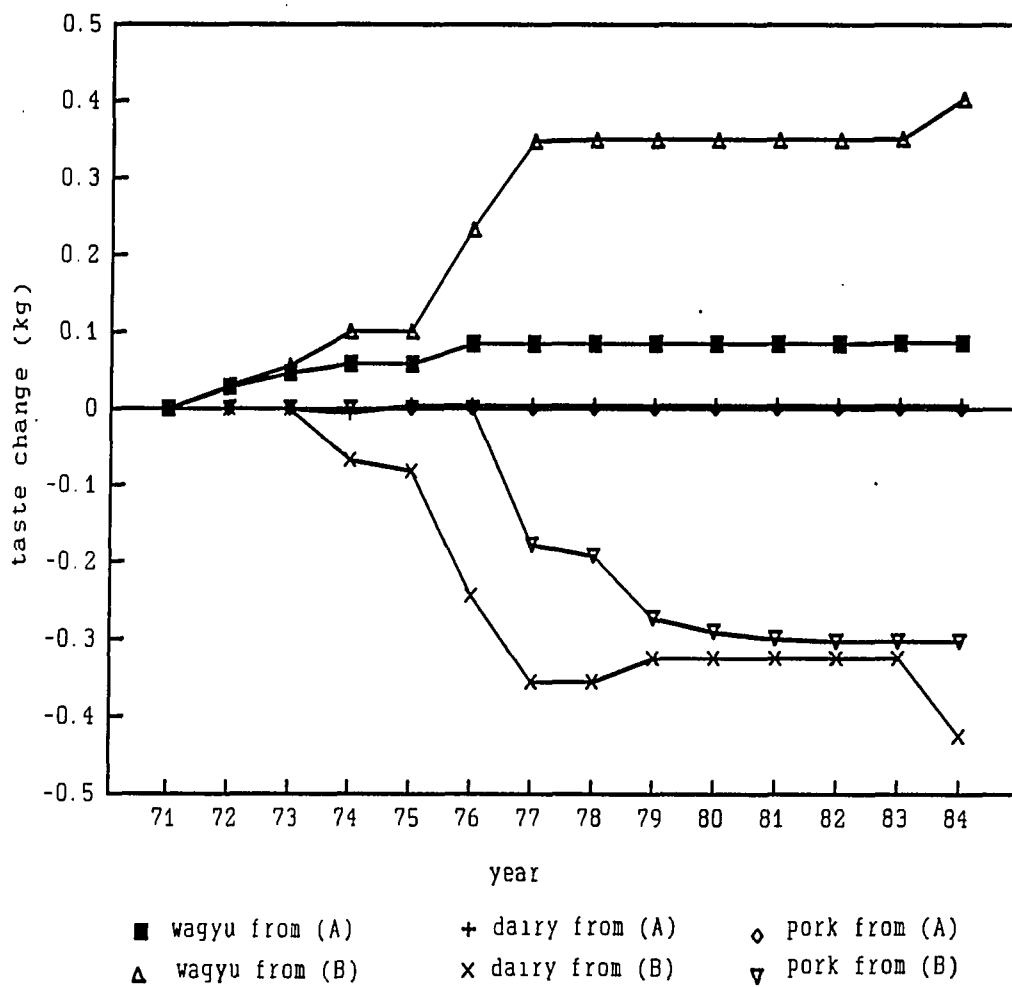
Figure 7. Taste change in Japanese meat demand
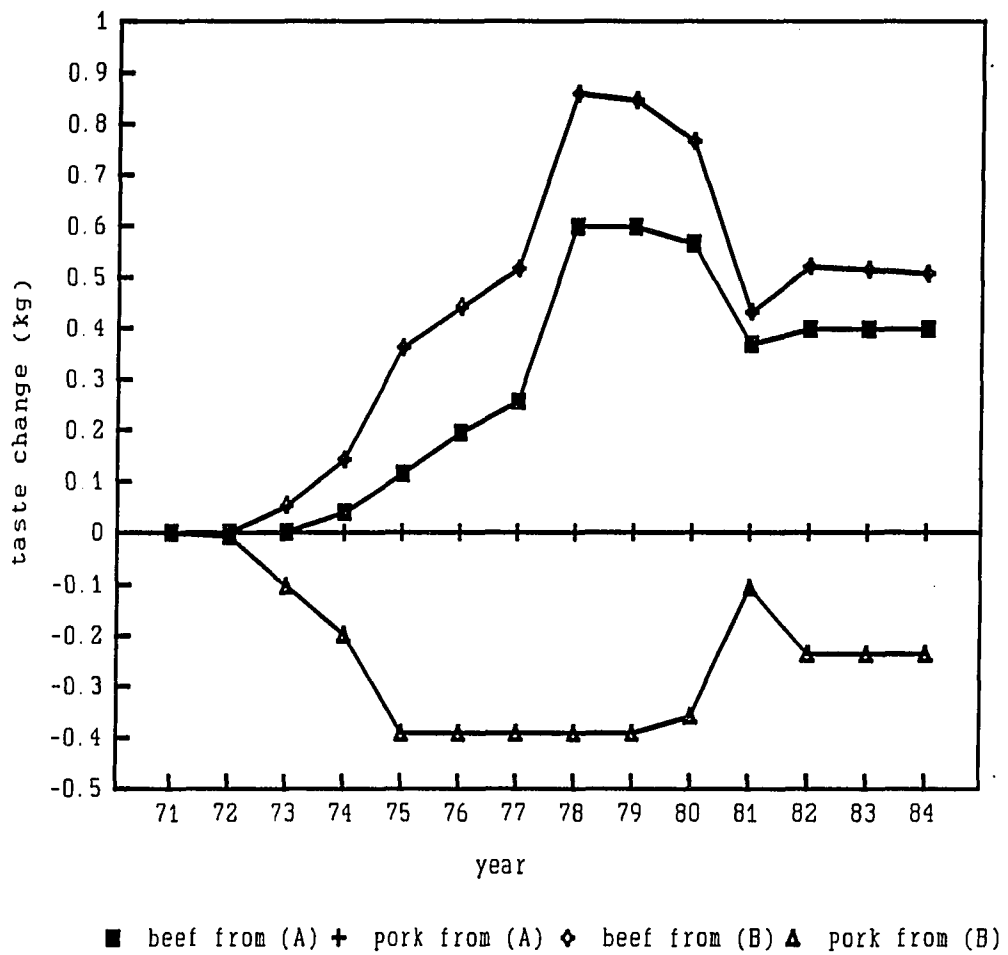
Figure 8. Taste change in South Korean meat demand

that occurs almost 10 years later in the United States.

The results for Japan show a positive movement toward native Japanese, or Wagyu, beef and a negative movement against Japanese dairy beef and imported beef. The magnitude of these changes is very small but represents a significant proportion of consumption. (In 1984, Japanese consumers ate 1.089 kg of Wagyu beef and 2.751 kg of dairy beef.) There is evidence of a slight shift away from pork in Japan while all the chicken data were consistent with preferences. Wahl, Hayes, and Williams report that Japanese farmers replaced Wagyu draft animals with tractors in the early 1970s and began fattening Wagyu animals for beef production. This change means that the quality of Wagyu beef would have improved considerably during this period. In the Japanese government statistics, data for Wagyu animals do not differentiate between retired draft animals and younger custom-fed beef animals. It seems likely, therefore, that the source of the inconsistency in Japan was data driven rather than consumer driven.

The South Korean results indicate a positive movement toward beef up to 1978, followed by a slight decrease to 1984. A slight movement against pork occurred between 1972 and 1976, but this was almost reversed in 1981. Again, no violations in the chicken data were detected.

CONCLUSIONS

This paper presents a way of detecting violation of consumer preference theory that has some of the advantages of existing parametric and nonparametric methods. The proposed method does not require any subjective input on behalf of the modeler and is therefore less subject to pretesting and data mining. The new method has the ability to detect slight violations in preferences, even when the budget constraint has shifted out, a feature that has not been found in the nonparametric models that have been presented to date.

The advantages of the new method are introduced by adding additional information about consumer behavior to previously available non-parametric methods. We assume that consumers meet their budget constraints, and that additional expenditure, or income, does not cause demand for any good to fall. We show that if one is prepared to accept additional restrictions on the magnitude or rate of change of expenditure elasticities that the sensitivity of the test is improved.

The model was used to examine meat demand data for the United States, Canada, Japan, and South Korea. The results indicate that a shift away from beef has occurred in the United States and Canada, while the opposite may have been true in Japan and South Korea. Smaller negative shifts have occurred against pork in all four countries. U.S. and especially Canadian consumers seem to have moved toward chicken, whereas Japanese and South Korean consumers have remained neutral.

The methodology used here has many possible applications. For example, one could determine whether generic or branded advertizing campaigns have been successful. One could also measure the impact of societal changes on demand for commodities or commodity aggregates. Finally, one could assume that consumer preferences are constant and check for structural change before using data for econometric purposes.

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PART II. EXPECTED UTILITY MAXIMIZATION WITH TRUNCATED
         DISTRIBUTIONS: AN APPLICATION OF THE MEAN VALUE
         THEOREM

INTRODUCTION

Losq (1982) used a mean value theorem to examine expected utility maximizing hedging behavior in the presence of both price and production uncertainty. This paper introduces options markets into Losq's model. Options truncate the price distribution at the strike price, and therefore invalidate the required conditions for the application of mean variance analysis. Our results show that the availability of options markets may cause behavior that violates Losq's results. We also show that the mean variance analysis will be misleading if production process is stochastic and/or if futures and options prices are perceived to be unbiased.

Other institutions such as insurance and some government programs truncate the distribution of price or revenue. The application of the mean value theorem presented here is therefore useful for situations other than that considered here.

MODEL

One can replicate the payoff of any combination of futures, puts, and calls with any two of these assets. Therefore, we will focus on futures and put options. For simplicity, assume that production variability in one region does not affect prices,[1] that a producer makes hedging decision after deciding on inputs levels, that only one strike price for put options is available, and that this is the current futures price. The random profit at harvest time can be written as

$$\tilde{Y} = \tilde{P}\tilde{Q} + (F-\tilde{F})X + (R - \tilde{R})Z - C(\bar{Q}) \tag{1}$$

where subscripts $\sim$ and $-$ denote a random variable and expected value, respectively; $\tilde{Q}$ is the random output; X and Z are the futures and put options sold by the producer, respectively; $C(\bar{Q})$ is a cost function; F is the futures price at the time of production decision; $\tilde{F}$ is a futures price at harvest; $\tilde{P}$ is the cash price at harvest; R is the put option price at the decision time; and $\tilde{R}$ is the terminal value of a put option with $\tilde{R} = (F - \tilde{F})L$, where $L = 1$ if $\tilde{F} \leq F$ and $L = 0$ if $\tilde{F} \geq F$.

Following Benninga, Eldor, and Zilcha (1983, 1984) and Lapan, Moschini, and Hanson (1990),[2] the cash price is assumed to be a linear regressive function of the futures price:

$$\tilde{P} = \tau + \tilde{\beta} + \tilde{\epsilon}$$

where $E[\tilde{\epsilon}] = 0$ and $\tilde{\epsilon}$ is assumed to be independent of $\tilde{F}$ and $\tilde{Q}$.

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[1] Results for the dependence case are available in Part 3.

[2] Lapan, Moschini, and Hanson introduce options into an expected utility maximization problem where production is certain. The absence of production uncertainty in their model allows for a more straight forward method than the mean value theorem used here and by Losq.

The producer is assumed to choose X and Z to maximize the expected utility of random

profit, that is, $\underset{X, Z}{\text{Max}}$ $E[u(\tilde{Y})]$, where u is a utility function and is assumed that $u' > 0$, $u'' < 0$,

and $u''' > 0$. Assuming that $F = \bar{F}$ and $R = \bar{R}$, the first order conditions are

$$E[u'(\tilde{Y})(\bar{F} - \tilde{F})] = 0 \quad or \quad E[u'(\tilde{Y})(\tilde{F} - \bar{F})] = 0 \qquad (2.1)$$

$$E[u'(\tilde{Y})\{\bar{R} - (\bar{F} - \tilde{F})L\}] = 0 \quad or \quad E[u'(\tilde{Y})\{(\tilde{F} - \bar{F})L + \bar{R}\}] = 0 \qquad (2.2)$$

SOLVING FOR OPTIMAL FUTURES AND OPTIONS DEMAND

By defining $g(\tilde{F}) = E[u'(\tilde{Y})|\tilde{F}]$ and using $E[\bullet] = E[E\{\bullet|\tilde{F}\}]$, the first order

conditions in (2.1) and (2.2) can be written as

$$E[u'(\tilde{Y})(\tilde{F} - \bar{F})] = E[g(\tilde{F})(\tilde{F} - FxR)] = 0 \qquad (3.1)$$

$$E[u'(\tilde{Y})\{(\tilde{F} - \bar{F})L + \bar{R}\}] = E[g(\tilde{F})\{(\tilde{F} - \bar{F})L + \bar{R}\}] = 0 \qquad (3.2)$$

The conditions for using the mean value theorem are continuity and differentiability.

With the existence of options, the price distribution is truncated at the strike price and thus $g(\tilde{F})$

is not differentiable at $\bar{F}$. That is, since $L = 1$ when $\tilde{F}$ approaches $\bar{F}$ from the left side and

$L = 0$ when $\tilde{F}$ approaches $\bar{F}$ from the right side, the slopes of $g(\tilde{F})$ with respect to $\tilde{F}$ at $\bar{F}^{+}$ and

$\bar{F}^{-}$ are

$$\lim_{\tilde{F} \to \bar{F}^{+}} \frac{\partial g(\tilde{F})}{\partial \tilde{F}} = E[u''(\tilde{Y})(\beta \bar{Q} - X)|\tilde{F} \to \bar{F}^{+}]$$

and

$$\lim_{\tilde{F} \to \bar{F}^{-}} \frac{\partial g(\tilde{F})}{\partial \tilde{F}} = E[u''(\tilde{Y})(\beta \bar{Q} - X + LZ)|\tilde{F} \to \bar{F}^{-}]$$

thus

$$if \quad Z \neq 0, \quad \lim_{\tilde{F} \to \bar{F}^{+}} \frac{\partial g(\tilde{F})}{\tilde{F}} \neq \lim_{\tilde{F} \to \bar{F}^{-}} \frac{\partial g(\tilde{F})}{\partial \tilde{F}}$$

Therefore, $g(\tilde{F})$ is not differentiable at $\bar{F}$ although it is continuous. However, $g(\tilde{F})$ is

differentiable over the intervals $[0, \bar{F}]$ and $[\bar{F}, \infty)$ and thus the mean value theorem can be

applied separately to $g(\tilde{F})$ in the regions $[0, \bar{F}]$ and $[\bar{F}, \infty)$.
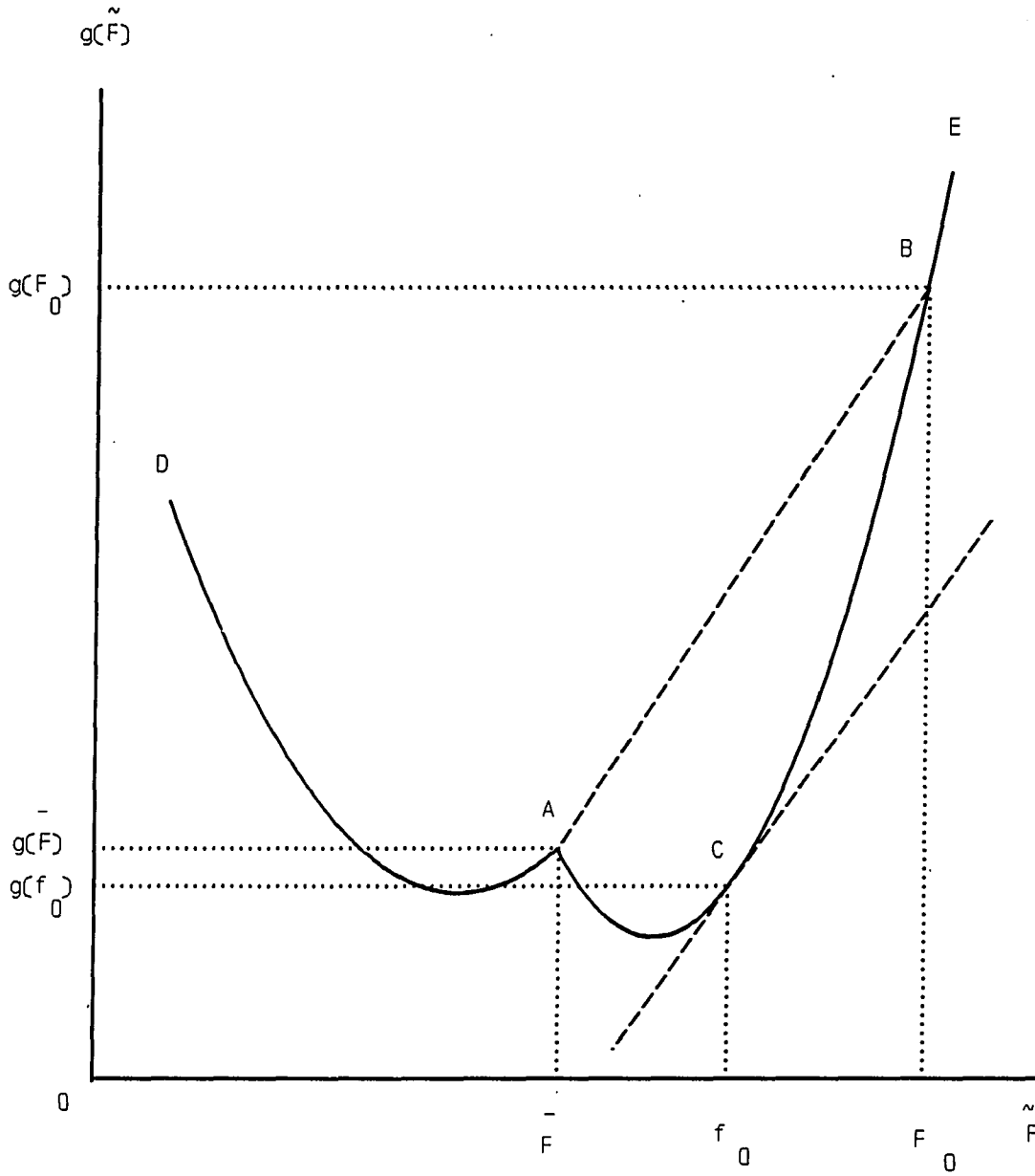
Figure 1. A schematic representation of how the expected marginal utility conditioned on realized futures price reponds to the futures price using the mean value theorem

Before proceeding, note that $g(\tilde{F})$ is strictly convex in $\tilde{F}$ over the interval $[0, \bar{F}]$ and

$[\bar{F}, \infty)$. That is, $\partial^2/g(\tilde{F})/\partial\tilde{F}^2$ $\{= E[u'''(\tilde{Y})(\beta\tilde{Q} - X + LZ)^2|\tilde{F}]\}$ is always positive in $[0, \bar{F}]$

and $[\bar{F}, \infty)$ if nonincreasing absolute risk aversion is assumed.[3]

Figure 1 shows how the mean value theorem can be applied to $g(\tilde{F})$ over the interval

$[\bar{F}, \infty)$. Suppose that curve DACBE represents $g(\tilde{F})$. Then, there is a futures price f_0 such that

$g'(f_0)$ equals the slope of the line AB. Here, for some F_0, f_0 is unique given the strict convexity

of $g(\tilde{F})$ in $[\bar{F}, \infty)$. Equivalently, the slope of the line connecting $[\bar{F}, g(\bar{F})]$ and $[\tilde{F}, g(\tilde{F})]$ for

any $\tilde{F}$ in $\tilde{F} > \bar{F}$ is the same as $g'(\tilde{f})$, that is,

$$\frac{g(\tilde{F}) - g(\bar{F})}{\tilde{F} - \bar{F}} = E[u''(\tilde{y})(\beta\tilde{Q} - X + LZ)|\tilde{F}] \tag{4}$$

where $\tilde{y}$ is profit associated with a futures price of $\tilde{f}$ and $\tilde{f}$ is monotonically increasing function

of $\tilde{F}$ because $g(\tilde{F})$ is strictly convex in $[\bar{F}, \infty)$. The left hand side in (4) represents the slope of

line AB and the right hand side represents the slope of $g(\tilde{F})$ at $\tilde{f}$. Applying the mean value

theorem to $g(\tilde{F})$ over the interval $[0, \bar{F}]$ requires a similar procedure.

From (4), $g(\tilde{F})$ is

$$g(\tilde{F}) = g(\bar{F}) + (\tilde{F} - \bar{F})E[u''(\tilde{y})(\beta\tilde{Q} - X + LZ)|\tilde{F}] \tag{5}$$

Substituting (5) into (3.1) gives

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[3] Under nonincreasing absolute risk aversion, $\partial A/\partial\tilde{Y} = -[u'''(\tilde{Y})/u'(\tilde{Y})] + [u''(\tilde{Y})/u'(\tilde{Y})]^2$ ≤ 0. This means that u''' must be positive. Therefore, the second derivative of $g(\tilde{F})$ with respect to $\tilde{F}$ is positive.

$$E[(\tilde{F} - \bar{F})g(\bar{F})] + E[(\tilde{F} - \bar{F})^2\{E[u''(\tilde{y})(\beta\tilde{Q} - X + LZ)|\tilde{F}]\}] = 0 \qquad (6)$$

Since $g(\bar{F})$ is a fixed number and $E[E\{\bullet|\tilde{F}\}] = E[\bullet]$, (6) can be rewritten as

$$E[(\tilde{F} - \bar{F})]g(\bar{F}) + E[u''(\tilde{y})(\tilde{F} - \bar{F})^2(\beta\tilde{Q} - X + LZ)] = 0$$

Since $E[\tilde{F} - \bar{F}] = 0$, equation (3.1) can be rewritten as follows:

$$E[u''(\tilde{y})(\tilde{F} - \bar{F})^2(\beta\tilde{Q} - X + LZ)] = 0 \qquad (7)$$

Equation (3.2) can be rewritten in a similar manner with (6):

$$\alpha E_1[g(\tilde{F})(\tilde{F} - \bar{F})] + \bar{R}E[u'(\tilde{Y})] = 0 \qquad (8)$$

where $\alpha = \text{Prob}[\tilde{F} \le \bar{F}]$ and subscript 1 represents the conditional expectation on $\tilde{F} \le \bar{F}$, that is, $E_1[\bullet] = E[\bullet|\tilde{F} \le \bar{F}]$.

By substituting $g(\tilde{F})$ into (5), the first term (FT) in the left hand side of equation (8) can be written as

$$\alpha E_1[(\tilde{F} - \bar{F})\{g(\bar{F}) + (\tilde{F} - \bar{F})E[u''(\tilde{y})(\beta\tilde{Q} - X + LZ)|\tilde{F}]\}]$$

Factoring out terms within $\{\bullet\}$, and using $E[E\{\bullet|\tilde{F}\}] = E[\bullet]$ and $L = 1$ when $\tilde{F} \le \bar{F}$, this term becomes

$$\alpha g(\bar{F})E_1[\tilde{F} - \bar{F}] + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2(\beta\tilde{Q} - X + Z)]$$

Now using $\bar{R} = E[(\tilde{F} - \bar{F})L] = -\alpha E_1[\tilde{F} - \bar{F}]$ and $g(\bar{F}) = E[u'(\tilde{Y})|\tilde{F} = \bar{F}]$ we get

$$\alpha E_1[g(\tilde{F})(\tilde{F} - \bar{F})] = -\bar{R}E[u'(\tilde{Y})|\tilde{F} = \bar{F}] + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2(\beta\tilde{Q} - X + Z)]$$

Therefore, (8) can be rewritten as

$$\bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y})|\tilde{F} = \bar{F}]\} + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2(\beta\tilde{Q} - X + Z)] = 0 \qquad (9)$$

Factoring out terms in $\{\bullet\}$, (7) and (9) can be rewritten, at the optimum, as follows

$$\beta E[u''(\tilde{y})(\tilde{F} - \bar{F})^2\tilde{Q}] - X^*E[u''(\tilde{y})(\tilde{F} - \bar{F})^2]$$
$$+ Z^*\alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2] = 0 \qquad (10.1)$$

$$\bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y})|\tilde{F} = \bar{F}]\} + \beta\alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2\tilde{Q}]$$
$$- X^*\alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2] + Z^*\alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2] = 0 \qquad (10.2)$$

where X^* and Z^* represent the optimal futures and put options amounts, respectively.

Consequently, equations (10.1) and (10.2) can be rearranged as follows:

$$\mathcal{L}_{FF}X^* - \mathcal{L}_{FFl}Z^* = \beta a \qquad (11.1)$$

$$- \mathcal{L}_{FFl}X^* + \mathcal{L}_{FFl}Z^* = -\beta b + c \qquad (11.2)$$

where $\mathcal{L}_{FF} = E[u''(\tilde{y})(\tilde{F} - \bar{F})^2]$, $\mathcal{L}_{FFl} = \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2]$, $a = E[u''(\tilde{y})\tilde{Q}(\tilde{F} - \bar{F})^2]$, b $= \alpha E_1[u''(\tilde{y})\tilde{Q}(\tilde{F} - \bar{F})^2]$, c $= -\bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y})|\tilde{F} = \bar{F}]\}$.

By assumption $u'' < 0$, therefore $\mathcal{L}_{FF}$, $\mathcal{L}_{FFl}$, a, and b are negative. Also, $\mathcal{L}_{FF} <$

$\mathscr{L}_{FF1} < 0$ and $a < b < 0$.[4]

To figure out the sign of c is less straightforward. Using $c = -\bar{R}[E\{g(\tilde{F})\} - E\{g(\bar{F})\}] = -\bar{R}E[g(\tilde{F}) - g(\bar{F})]$ and substituting $g(\tilde{F}) - g(\bar{F}) = (\tilde{F} - \bar{F})E[u''(\tilde{y})(\beta\tilde{Q} - X^* + LZ^*)|\tilde{F}]$, c can be rewritten as

$$c = -\bar{R}E[u''(\tilde{y})(\beta\bar{Q} - X^* + LZ^*)]$$
$$= -\bar{R}Cov[\tilde{F}, u''(\tilde{y})(\beta\bar{Q} - X^* + LZ^*)] \tag{12}$$

The covariance term has the same sign as $\partial[u''(\tilde{y})(\beta\tilde{Q} - X^* + LZ^*)]/\partial\tilde{F} = u'''(\tilde{y})(\beta\tilde{Q} - X^* + LZ^*)^2(\partial\tilde{f}/\partial\tilde{F})$, which is positive under the nonincreasing absolute risk aversion. Therefore, c is also negative.

From (11.1) and (11.2), X^* and Z^* can be obtained as

$$X^* = \frac{\{\beta(a - b) + c\}\mathscr{L}_{FF1}}{\Delta} \tag{13.1}$$

and

$$Z^* = \frac{\beta(\mathscr{L}_{FF1}a - \mathscr{L}_{FF}b) + \mathscr{L}_{FF}c}{\Delta} \tag{13.2}$$

where $\Delta = \mathscr{L}_{FF}\mathscr{L}_{FF1} - \mathscr{L}_{FF1}^2 = \mathscr{L}_{FF1}(\mathscr{L}_{FF} - \mathscr{L}_{FF1}) > 0$ since $\mathscr{L}_{FF} < \mathscr{L}_{FF1} < 0$. Consequently, the producer always sells futures, that is, $X^* > 0$. On the other hand, whether he or she sells or buys put options is ambiguous.

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[4] Because $\mathscr{L}_{FF} - \mathscr{L}_{FF1} = (1 - \alpha)E[u''(\tilde{y})(\tilde{F} - \bar{F})^2|\tilde{F} \geq \bar{F}] < 0$, $\mathscr{L}_{FF} < \mathscr{L}_{FF1} < 0$. Similarly, $a - b = (1 - \alpha)E[u''(\tilde{y})\tilde{Q}(\tilde{F} - \bar{F})^2|\tilde{F} \geq \bar{F}] < 0$ and thus $a < b < 0$.

Proposition 1

When both futures and options are available and when they are perceived to be unbiased, the optimal hedge for a firm with stochastic production may be less than, equal to, or greater than the nonstochastic optimal hedge $\beta\bar{Q}$.

Substituting $\tilde{Q} = \bar{Q} + (\tilde{Q} - \bar{Q})$ into a and b, we get

$$a = \bar{Q}\mathcal{L}_{FF} + \mathcal{L}_{QFF} \quad and \quad b = \bar{Q}\mathcal{L}_{FFI} + \mathcal{L}_{QFFI} \tag{14}$$

where $\mathcal{L}_{QFF} = E[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})^2]$ and $\mathcal{L}_{QFFI} = \alpha E_1[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})^2]$.

Substituting a and b obtained here into (13.1), X^* can be rewritten as

$$X^* = \beta\bar{Q} + \frac{\mathcal{L}_{FFI}\{\beta(\mathcal{L}_{QFF} - \mathcal{L}_{QFFI}) + c\}}{\Delta} \tag{15}$$

where $\mathcal{L}_{QFF} = E[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})^2] = E[(\tilde{F} - \bar{F})^2 Cov\{u''(\tilde{y}), \tilde{Q} | \tilde{F}\}]$. The conditional covariance has the same sign as $\partial u''(\tilde{y})/\partial\tilde{Q} = \beta u'''(\tilde{y})\tilde{f}(\partial\tilde{f}/\partial\tilde{F})$, which is positive since $u''' > 0$ and $\partial\tilde{f}/\partial\tilde{F} > 0$ with nonincreasing absolute risk aversion. Similarly, $\mathcal{L}_{QFFI}$ is positive but is less than $\mathcal{L}_{QFF}$, that is, $0 < \mathcal{L}_{QFFI} < \mathcal{L}_{QFF}$. The second term of the right hand side in (15) can be positive, zero, or negative. Therefore, the optimal futures amount sold by the producer can be less than, equal to, or greater than $\beta\bar{Q}$. *QED*

McKinnon (1967) and Losq showed that in the absence of options markets production uncertainty causes producers to hedge less than would otherwise be the case ($X^* < \beta\bar{Q}$). When the options market is introduced here, the results indicate that X^* can be less than, equal to, or greater than $\beta\bar{Q}$.

This result can be supported by numerical simulation. Assume that $\tilde{P} = \tilde{F}$ and that the

producer exhibits a constant absolute risk aversion, that is, $u(\widetilde{Y}) = -\exp[-A\widetilde{Y}]$, where A is the constant absolute risk-aversion coefficient. Also, assume that price is normally distributed with mean \$7.32 and variance 2.48 and that output is normally distributed with mean 3,000 and variance 5,300.[5] The results obtained from the expected utility maximization are $X^* = 3,140$ and $Z^* = 350$ for $A = 0.00015$ and $X^* = 2,870$ and $Z^* = 70$ for $A = 0.00045$. Because $\widetilde{P} = \widetilde{F}$, $\beta\bar{Q} = 3,000$. Therefore, X^* is greater than 3,000 bushels when $A = 0.00015$ and less than 3,000 bushels when $A = 0.00045$.

Proposition 2

If options and futures prices are unbiased and if there is no production uncertainty, optimal behavior for expected utility maximizing firms is the same as for firms that maximize a mean variance utility function. This relationship does not hold when production uncertainty is introduced.

Becuase options truncate the price distribution, mean variance analysis can be used only when the utility function is quadratic, that is, $u''' = 0$.

Substituting a and b in (14) into (13.2), Z^* can be rewritten as

$$Z^* = 0 + \frac{\beta(\mathcal{L}_{FFI}\mathcal{L}_{QFF} - \mathcal{L}_{FF}\mathcal{L}_{QFFI}) + \mathcal{L}_{FF}c}{\Delta} \tag{16}$$

Substituting $\widetilde{Q} = \bar{Q} + (\widetilde{Q} - \bar{Q})$ into c in (12) and rearranging, one obtains

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[5] These numbers except mean output are based on data for Iowa soybean growers (Soybeans: Iowa's Premier Crop).

$$c = -\bar{R}[\beta \bar{Q} \mathscr{L}_F + \mathscr{L}_{QF} - X^* \mathscr{L}_F + Z^* \mathscr{L}_{FI}] \qquad (17)$$

where $\mathscr{L}_F = E[u''(\tilde{y})(\tilde{F} - \bar{F})]$, $\mathscr{L}_{FI} = \alpha E_1[u''(\tilde{F} - \bar{F})]$, and $\mathscr{L}_{QF} = E[u''(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})]$. Substituting X^* and Z^* from (15) and (16) into (17) and rearranging yields

$$c = \frac{\bar{R}[\beta \Delta \mathscr{L}_{QF} - \beta \mathscr{L}_F \mathscr{L}_{FFI}(\mathscr{L}_{QFF} - \mathscr{L}_{QFFI}) + \beta \mathscr{L}_{FI}(\mathscr{L}_{FFI}\mathscr{L}_{QFF} - \mathscr{L}_{FF}\mathscr{L}_{QFFI})]}{\bar{R}(\mathscr{L}_F \mathscr{L}_{FFI} - \mathscr{L}_{FI}\mathscr{L}_{FF}) - \Delta}$$

Suppose that production process is nonstochastic. Then, $\mathscr{L}_{QF}$, $\mathscr{L}_{QFF}$, and $\mathscr{L}_{QFFI}$ are zero and $c = 0$ and thus the second terms of right hand side in (15) and (16) are zero. The producer will sell $\beta \bar{Q}$ in futures markets and options are redundant. This result does not depend upon the sign of u'''.

The signs of c, $\mathscr{L}_{QFF}$, and $\mathscr{L}_{QFFI}$ depend upon the sign of u'''. We have already singed these terms when $u''' > 0$ in statements followed by (12) and (15), that is, c is opposite in sign to u''', and $\mathscr{L}_{QFF}$ and $\mathscr{L}_{QFFI}$ have the same sign as u'''.[6] Suppose that $u''' = 0$. Then, these terms are zero and thus the second term of the right hand side in (15) and (16) are zero. Therefore, The optimal futures and put options amounts are $\beta \bar{Q}$ and 0, respectively. *QED*

If one is willing to assume the unbiased futures and options prices and nonstochastic production, the mean variance analysis will provide the same result as expected utility maximization. However, if production is stochastic, the results obtained from the mean variance analysis will be misleading. For example, suppose that output is normally distributed with mean 20,000 bushels and variance 2,300,000 and that price distribution is the same as before. Then,

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[6] When $u''' < 0$ these results are reversed.

the optimal futures and put options amounts are 15,370 and −460 for A = 0.00045 and 21,690

and 5,650 for A = 0.00001. The mean variance solution in both of these cases is to sell 20,000

on the futures market and not to participate in the options market.

CONCLUSIONS

This paper examines behavior of an expected utility maximizing individual who faces both price and production uncertainty and who has access to both futures and options markets. The key insight is that mean value theorem can be used to solve expected utility maximization problems even when the price distribution is truncated. The results show that firms will almost always use options and the firm will hedge more or less in the futures market than it would in the absence of production uncertainty.

The results also show that mean variance produces accurate results so long as markets are perceived to be unbiased and if there is no production uncertainty. The error caused by the improper use of mean variance analysis when production uncertainty exists can be quite large.

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PART III. HEDGING PRODUCTION RISK WITH OPTIONS

INTRODUCTION

In a recent paper, Lapan, Moschini, and Hanson (LMH) extended Sandmo's expected utility model to analyze production, hedging, and speculative decisions when futures and options markets exist. One important implication of this work is that when individuals perceive futures and options markets to be unbiased and when cash prices are a linear function of futures prices, there is no place for options as hedging instruments.

The key to the LMH result is that one can divide the price risk into a component attributable to changes in end-of-period futures prices and an orthogonal component reflecting undiversifiable basis risk. Because the diversifiable risk is linear in futures prices, futures contracts (which are linear in futures prices) dominate options contracts, which are nonlinear in futures prices.

A recent survey of Iowa farmers indicated that as many producers use options to hedge as use futures (Sapp). This finding is in contrast to the LMH result and raises the question of the conditions under which producers may find it optimal to use options to hedge. In the context of the LMH result, this is equivalent to the conditions under which the risk faced by producers is nonlinear in futures prices. One way to introduce this nonlinearity is to introduce production uncertainty. For example, grain producers may believe that low individual grain yields (caused by drought) are associated with high grain prices. If producers sell more grain on the futures or forward markets than they obtain from on-farm production, they will be forced to purchase expensive grain to meet contractual commitments. Alternatively, if prices are low (due possibly to abundant rainfall in the Upper Midwest) and if farm production is greater than anticipated, producers may not hedge enough production to eliminate all the price risk.

In the case in which the expected correlation between local output and futures prices is zero, one can still develop an intuitive motivation for options because the effect of quantity uncertainty on profit is greater at higher prices. For example, a producer may anticipate price changes and fully hedge a 100 bushels/acre crop on the futures market. If actual production is only 80 bushels/acre, the producer will be exposed to a loss that increases with increases in the futures price. At $5/bushel this loss amounts to $100/acre and at $10/bushel this loss is $200/acre. This somewhat counterintuitive situation occurs because losses in the fully hedged futures position more than offset the benefits of increasing prices on the physical position.

The purposes of this paper are to introduce production uncertainty into the LMH model both theoretically and by means of simulation examples and to show how options can be used to hedge against production uncertainty when output is uncertain. We focus on investors who believe that both futures and options are unbiased.

This paper is organized as follows. The model is set out under the assumption that local production variation does not affect the price of the commodity, and the optimal positions for futures and options are illustrated. The effect of production uncertainty on optimal hedging behavior is then emphasized. Next, the independence assumption is relaxed.

As might be expected with random price and output variables, two financial instruments, and a truncated distribution, the results for the general case require a lengthy and somewhat tedious derivation. One motivation for the presentation of these derivations is that they can be used as the basis for a more specific and richer analysis, which is demonstrated in the penultimate section of the paper by simulating the decision-making process of an Iowa corn producer. The final section presents the conclusions of the analysis.

MODEL

It is possible to replicate the payoff of any combination of futures, puts, and calls with any two of these three assets. Our attention will focus on futures and put options. For simplicity, only one strike price for put options is considered and this strike price is assumed to be the current futures price. Suppose that a producer makes hedging decisions after deciding input levels. Also suppose initially that production variability in one region does not affect prices. The random profit at harvest can be written as

$$\tilde{Y} = \tilde{P}\tilde{Q} + (F - \tilde{F})X + (R - \tilde{R})Z - C(\bar{Q}) \tag{1}$$

where the superscripts ~ and - denote a random variable and expected value, respectively; $\tilde{Q}$ is the random output; X and Z are the futures and put options sold by the producer, respectively; $C(\bar{Q})$ is a cost function; F is the futures price at the time of the production decision; $\tilde{F}$ is a futures price at harvest time; $\tilde{P}$ is the cash price at harvest time; R is the put option price at decision time; and $\tilde{R}$ is the terminal value of a put option with

$$\tilde{R} = (F - \tilde{F})L \quad where \begin{cases} L = 1 \ if \ \tilde{F} \leq F \\ \\ L = 0 \ if \ \tilde{F} \geq F \end{cases} \tag{2}$$

Following Benninga, Eldor, and Zilcha (1983, 1984) and LMH, the cash price is assumed to be a linear regressive function of the futures price:

$$\tilde{P} = \tau + \beta\tilde{F} + \tilde{\epsilon} \tag{3}$$

where $E[\tilde{\epsilon}] = 0$ and $\tilde{\epsilon}$ is assumed to be independent of $\tilde{F}$ and $\tilde{Q}$.[1] Substituting (2) and (3) into (1), the random profit yields

$$\tilde{Y} = (\tau + \beta\tilde{F} + \tilde{\epsilon})\tilde{Q} + (F - \tilde{F})X + \{R - (F - \tilde{F})L\}Z - C(\bar{Q}) \qquad (4)$$

The producer is assumed to choose X and Z to maximize the expected utility of random profit, that is, $\underset{X, Z}{\text{Max}}\ E[u(\tilde{Y})]$ where $u'(\tilde{Y}) > 0$ and $u''(\tilde{Y}) < 0$. Assuming that $F = E[\tilde{F}] = \bar{F}$ and $R = E[\tilde{R}] = \bar{R}$, the first-order conditions are

$$E[u'(\tilde{Y})(\bar{F} - \tilde{F})] = 0 \quad or \quad E[u'(\tilde{Y})(\tilde{F} - \bar{F})] = 0 \qquad (5.1)$$

and

$$E[u'(\tilde{Y})\{\bar{R} - (\bar{F} - \tilde{F})L\}] = 0 \quad or \quad E[u'(\tilde{Y})\{(\tilde{F} - \bar{F})L + \bar{R}\}] = 0 \qquad (5.2)$$

By defining $g(\tilde{F}) = E[u'(\tilde{Y})|\tilde{F}]$ and using $E[\bullet] = E[E\{\bullet|\tilde{F}\}]$, the first-order conditions in (5.1) and (5.2) can be written as:

$$E[u'(\tilde{Y})(\tilde{F} - \bar{F})] = E[(\tilde{F} - \bar{F})E\{u'(\tilde{Y})|\tilde{F}\}] = E[g(\tilde{F})(\tilde{F} - \bar{F})] = 0 \qquad (6.1)$$

and

$$E[u'(\tilde{Y})\{(\tilde{F} - \bar{F})L + \bar{R}\}] = E[\{(\tilde{F} - \bar{F})L + \bar{R}\}E\{u'(\tilde{Y})|\tilde{F}\}]$$
$$= E[g(\tilde{F})\{(\tilde{F} - \bar{F})L + \bar{R}\}] = 0 \qquad (6.2)$$

Given subjective distributions of prices and output and a known utility function, the optimal futures and put options position, denoted by X^* and Z^*, can be found by numerical optimization. However, because the true utility function and price and output distributions are not

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[1] We allow for basis risk in the theoretical model but assume it away in the simulation.

known, results derived from particular examples may be misleading and lack generality.

The mean value theorem[2] can be used to obtain some results for the general case where neither the utility function nor the price and output distributions are known. Using this theorem, one can solve for the signs of X^* and Z^* and/or the relative sizes of X^* and Z^*. In the general case, it can be shown that options are almost always used to hedge production uncertainty in a way that makes intuitive sense.

Using the mean value theorem, we can find a unique $\tilde{f}$ somewhere between $\tilde{F}$ and $\overline{F}$ such that $g(\tilde{F}) = g(\overline{F}) + (\tilde{F} - \overline{F}) g'(\tilde{f})$, where $g'(\tilde{f}) = E[u''(\tilde{y})(\beta\tilde{Q} - X + LZ)|\tilde{F}]$,[3] $\partial\tilde{f}/\partial\tilde{F} > 0$, and $\tilde{y}$ is the profit of an individual firm when the futures price is $\tilde{f}$. This procedure is shown in Appendix A. Substituting $g(\overline{F}) + (\tilde{F} - \overline{F})g'(\tilde{f})$ for $g(\tilde{F})$ in (6.1) and (6.2) and arranging, the following relations for X^* and Z^* can be obtained[4]

$$\mathscr{L}_{FF}X^* - \mathscr{L}_{FF1}Z^* = \beta a \qquad (7.1)$$

$$-\mathscr{L}_{FF1}X^* + \mathscr{L}_{FF1}Z^* = -\beta b + c \qquad (7.2)$$

where $\mathscr{L}_{FF} = E[u''(\tilde{y})(\tilde{F} - \overline{F})^2]$, $\mathscr{L}_{FF1} = \alpha E_1[u''(\tilde{y})(\tilde{F} - \overline{F})^2]$, $a = E[u''(\tilde{y})\tilde{Q}(\tilde{F} - \overline{F})^2]$, $b = \alpha E_1[u''(\tilde{y})\tilde{Q}(\tilde{F} - \overline{F})^2]$, $c = -\overline{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y})|\tilde{F} = \overline{F}]\}$, $\alpha = \text{Prob}[\tilde{F} \leq \overline{F}]$, and subscript 1 represents the conditioning on $\tilde{F} \leq \overline{F}$, that is, $E_1[\bullet] = E[\bullet|\tilde{F} \leq \overline{F}]$.

By assumption, $u''(\tilde{y})$ is negative for all $\tilde{y}$. It is obvious that $\tilde{Q}$ and $(\tilde{F} - \overline{F})^2$ are

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[2] Refer to Stein or Rudin.

[3] The term $g'(\tilde{f})$ is the expectation of $u''(\tilde{y})(\beta\tilde{Q} - X + LZ)$ conditioned on $\tilde{f}$. We can replace $g'(\tilde{f})$ with the conditional expectation of $u''(\tilde{y})(\beta\tilde{Q} - X + LZ)$ on $\tilde{F}$ because $\tilde{f}$ has a one-to-one relationship with $\tilde{F}$, and because $\tilde{f} \leq (\geq) \overline{F}$ if and only if $\tilde{F} \leq (\geq) \overline{F}$ (refer to Appendix A).

[4] The derivations of (7.1) and (7.2) are available in Appendix C.

positive, and therefore $\mathscr{L}_{FF}$, $\mathscr{L}_{FF1}$, a, and b are always negative. Also, we know that $\mathscr{L}_{FF} <$ $\mathscr{L}_{FF1} < 0$ and $a < b < 0$.[5]

Obtaining the sign of c is less straightforward. Using $c = -\bar{R}[E\{g(\tilde{F})\} - E\{g(\bar{F})\}] = -\bar{R}E[g(\tilde{F}) - g(\bar{F})]$ and substituting $g(\tilde{F}) - g(\bar{F}) = (\tilde{F} - \bar{F})E[u''(\tilde{y})(\beta\tilde{Q} - X^* + LZ^*)|\bar{F}]$, c can be rewritten as

$$c = -\bar{R}E[u''(\tilde{y})(\tilde{F} - \bar{F})(\beta\tilde{Q} - X^* + LZ^*)]$$
$$= -\bar{R}Cov[\tilde{F}, u''(\tilde{y})(\beta\tilde{Q} - X^* + LZ^*)]$$

(8)

The covariance term has the same sign as $\partial[u''(\tilde{y})(\beta\tilde{Q} - X^* + LZ^*)]/\partial\tilde{F} = u'''(\tilde{y})(\beta\tilde{Q} - X^* + LZ^*)^2(\partial\tilde{f}/\partial\tilde{F})$, which is positive under nonincreasing absolute risk aversion.[6] Therefore, c is also negative.

At the optimum, $\mathscr{L}_{FF}$, $\mathscr{L}_{FF1}$, a, b, and c are fixed numbers and (7.1) and (7.2) implicitly contain X^* and Z^*. Holding all values at the optimum, X^* and Z^* can be obtained from (7.1) and (7.2) as

$$X^* = \frac{\{\beta(a - b) + c\}\mathscr{L}_{FF1}}{\Delta}$$

(9.1)

and

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[5] $\mathscr{L}_{FF} - \mathscr{L}_{FF1} = (1 - \alpha)E[u''(\tilde{y})(\tilde{F} - \bar{F})^2|\tilde{F} \geq \bar{F}] < 0$ and $\mathscr{L}_{FF1} < 0$, therefore $\mathscr{L}_{FF} < \mathscr{L}_{FF1} < 0$. Similarly, $a - b = (1 - \alpha)E[u''(\tilde{y})\tilde{Q}(\tilde{F} - \bar{F})^2|\tilde{F} \geq \bar{F}] < 0$ and $b < 0$, which implies $a < b < 0$.

[6] Nonincreasing absolute risk aversion indicates that $u''' > 0$ (refer to Appendix A).

$$Z^* = \frac{\beta(\mathscr{L}_{FFI}a - \mathscr{L}_{FF}b) + \mathscr{L}_{FF}c}{\Delta} \tag{9.2}$$

where $\Delta = \mathscr{L}_{FF}\mathscr{L}_{FF1} - \mathscr{L}_{FFI}^2 = \mathscr{L}_{FF1}(\mathscr{L}_{FF} - \mathscr{L}_{FF1}) > 0$ because $\mathscr{L}_{FF} < \mathscr{L}_{FF1} < 0$,

$a < b < 0$, and $c < 0$. Consequently, the producer always sells futures, that is, $X^* > 0$.[7]

On the other hand, whether the producer sells or buys put options is ambiguous.

Before proceeding, we now focus on the additional hedging caused by production

uncertainty. It is useful to show that, with nonstochastic output, the optimal decision is to sell $\beta\bar{Q}$

on the futures market and to stay out of the options market.

Substituting $\tilde{Q} = \bar{Q} + (\tilde{Q} - \bar{Q})$ into c in (8) and rearranging yields

$$c = -\bar{R}\,[\beta\bar{Q}\mathscr{L}_F + \mathscr{L}_{QF} - X^*\mathscr{L}_F + Z^*\mathscr{L}_{FI}] \tag{10}$$

where $\mathscr{L}_F = E[u''(\tilde{y})(\tilde{F} - \bar{F})]$, $\mathscr{L}_{F1} = \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})]$, and $\mathscr{L}_{QF} = E[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})]$.

Substituting $\tilde{Q} = \bar{Q} + (\tilde{Q} - \bar{Q})$ into the expression for a and b, we get

$$a = \bar{Q}\mathscr{L}_{FF} + \mathscr{L}_{QFF} \quad and \quad b = \bar{Q}\mathscr{L}_{FF1} + \mathscr{L}_{QFF1}$$

where $\mathscr{L}_{QFF} = E[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})^2]$ and $\mathscr{L}_{QFF1} = \alpha E_1[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})^2]$.

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[7] The optimal futures and put option amounts of X^* and Z^* can be expressed as a function of $\mathscr{L}_{FF}$, $\mathscr{L}_{FF1}$, a, b, and c. In effect, these terms are also functions of X^* and Z^*. However, when X^* and Z^* are expressed in terms of $\mathscr{L}_{FF}$, $\mathscr{L}_{FF1}$, a, b, and c, which are fixed at the optimum, one can find the signs of X^* and Z^* and/or the relative sizes of X^* and Z^*. For example, suppose that $X^* = h_1(X^*, Z^*) > 0$, $Z^* = h_2(X^*, Z^*) > 0$, and $h_1(X^*, Z^*)$ is always greater than $h_2(X^*, Z^*)$. Then, one can conclude that $X^* > Z^* > 0$.

Substituting the values just obtained for a and b into (9.1) and (9.2), X^* and Z^* can be rewritten as

$$X^* = \beta\bar{Q} + \frac{\mathscr{L}_{FFI}\{\beta(\mathscr{L}_{QFF} - \mathscr{L}_{QFFI}) + c\}}{\Delta} \tag{11.1}$$

$$Z^* = 0 + \frac{\beta(\mathscr{L}_{FFI}\mathscr{L}_{QFF} - \mathscr{L}_{FF}\mathscr{L}_{QFFI}) + \mathscr{L}_{FF}c}{\Delta} \tag{11.2}$$

Substituting X^* and Z^* from (11.1) and (11.2) into (10) and rearranging, one obtains:

$$c = \frac{\bar{R}[\beta\Delta\mathscr{L}_{QF} - \beta\mathscr{L}_{F}\mathscr{L}_{FFI}(\mathscr{L}_{QFF} - \mathscr{L}_{QFFI}) + \beta\mathscr{L}_{FI}(\mathscr{L}_{FFI}\mathscr{L}_{QFF} - \mathscr{L}_{FF}\mathscr{L}_{QFFI})]}{\bar{R}(\mathscr{L}_{F}\mathscr{L}_{FFI} - \mathscr{L}_{FI}\mathscr{L}_{FF}) - \Delta}$$

When the production process is nonstochastic, $\mathscr{L}_{QF}$, $\mathscr{L}_{QFF}$, and $\mathscr{L}_{QFF1}$ are zero[8] and thus c = 0. The second terms in the right-hand sides of (11.1) and (11.2) are all zero, and thus the optimal futures amount under production certainty is $\beta\bar{Q}$ and options are redundant. Consequently, the right-hand sides of (11.1) and (11.2) can be separated into two parts: the first term in the right-hand side is the optimal futures and put options amounts sold by the producer under production certainty, and the second term represents the additional futures and put options amounts arising from production uncertainty.

Under independence of price uncertainty and output uncertainty, $\mathscr{L}_{QFF} = E[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})^2] = E[(\tilde{F} - \bar{F})^2 Cov\{u''(\tilde{y}), \tilde{Q} | \tilde{F}\}]$. The conditional covariance has the same sign

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[8] For example, $\mathscr{L}_{QF} = E[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})] = 0$ because $\tilde{Q} = \bar{Q}$ under nonstochastic production.
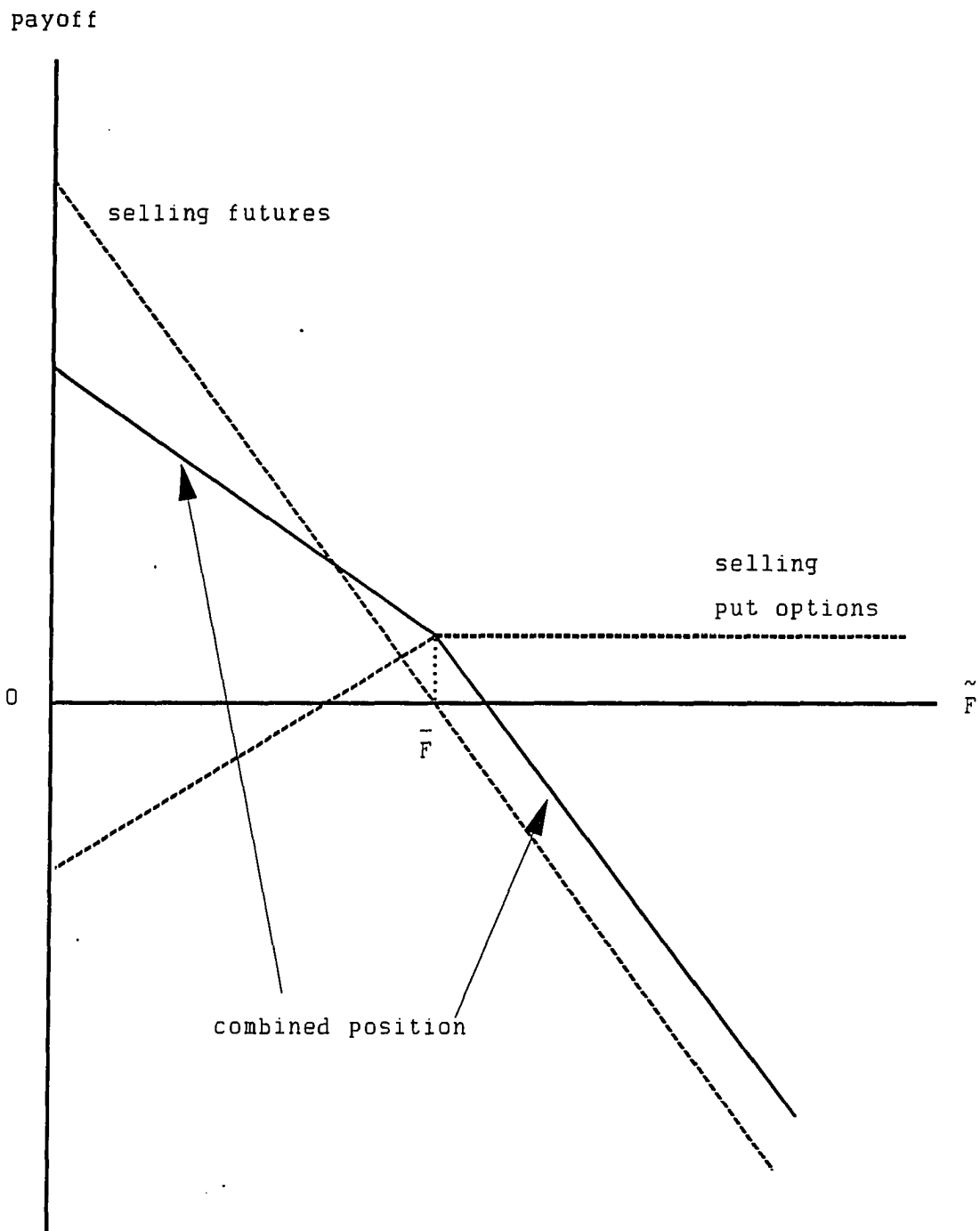
payoff

Figure 1. An example of the combined position of optimal futures and options

as $\partial u''(\tilde{y})/\partial \tilde{Q} = u'''(\tilde{y})\tilde{p}$ (where $\tilde{p} = \tau + \beta \tilde{f} + \tilde{\epsilon}$), which is positive because $u''' > 0$ with nonincreasing absolute risk aversion. Similarly, $\mathscr{L}_{QFF1}$ is positive. The additional futures and put options attributable to production uncertainty are denoted as ΔX^* and ΔZ^*, respectively. Then, subtracting ΔZ^* from ΔX^* yields

$$\Delta X^* - \Delta Z^* = \frac{(\mathscr{L}_{FF1} - \mathscr{L}_{FF})(c - \beta \mathscr{L}_{QFF1})}{\Delta} \qquad (12)$$

Because $\mathscr{L}_{FF1} - \mathscr{L}_{FF} > 0$, $c < 0$, $\mathscr{L}_{QFF1} > 0$, and $\Delta > 0$, the right-hand side of (12) is negative and one can conclude that $\Delta Z^* > \Delta X^*$, even though the signs of ΔX^* and ΔZ^* are unknown. Thus, production uncertainty (that is independent of prices) causes producers to take options and futures positions that are different from those taken when production is certain.

A standard "payoff" diagram is a good way to describe these positions. Figure 1 represents the profit or loss at harvest time in futures and options based on the futures price realized at harvest. If the producer has sold a futures contract, profits from that portion of the portfolio fall as the futures price at harvest increases because the producer has promised to deliver, at a fixed price, an asset whose value is increasing. The payoff from sale of a put option, on the other hand, will rise as the futures price rises from 0 to the strike price $(\bar{F})$. Because the put option has no value if $\tilde{F} \geq \bar{F}$, the payoff is independent of the realized futures price in the region from $\bar{F}$ to infinity. That the return to a put option is increasing in $\bar{F}$ is clear by observing that $\tilde{R}$ has a negative effect on profits in (1), but that $\bar{F}$ also has a negative effect on $\tilde{R}$ in (2). For example, if we know that the producer has sold more futures contracts than put options, we can describe how the total position responds to the realized futures price at harvest.
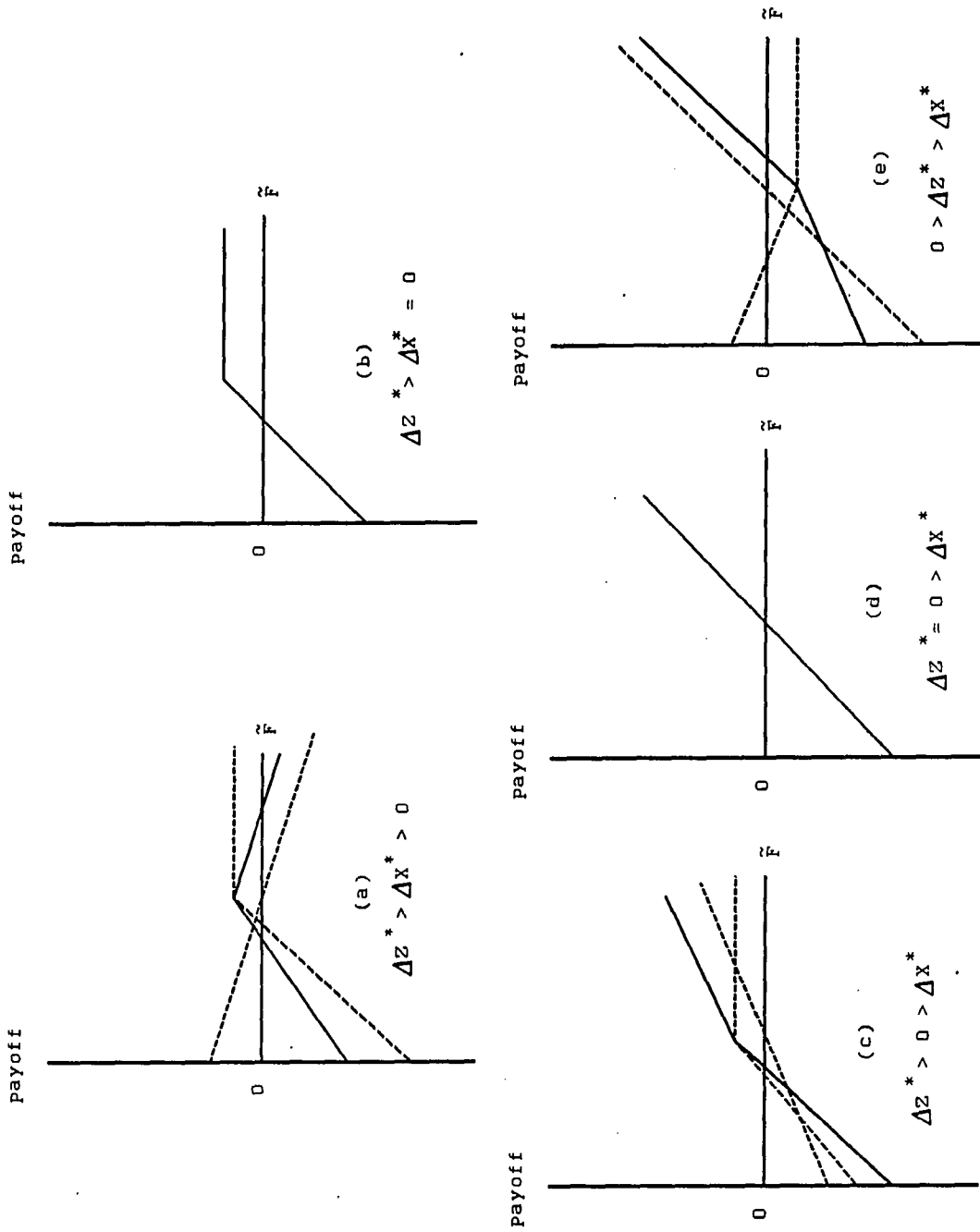
Figure 2. Additional hedging positions taken because of production uncertainty

The payoff line for the combined position is determined by adding the payoffs of the two assets vertically for each futures price realized at harvest time.

From Figure 1, we see that, for this particular example, at any point up to the strike price the reduced profit in the futures market from an increase in the futures price is greater than the increasing benefit (reduced loss) in the options market. At any point beyond the strike price, a price increase causes the loss in futures to increase, whereas the put options profit is not affected by realized futures price changes. Therefore, we can conclude that the net payoff is decreasing in realized futures price in all regions.

The precise nature of the additional positions will depend on the producer's utility function and the subjective distributions of output and prices. Nevertheless, only five possible outcomes can occur that satisfy (12). These are shown in (a) through (e) of Figure 2. The dotted lines represent the payoff diagrams in futures and put options and the continuous line represents the payoff for the combined position. Underhedging when the futures price is low is common to all five possibilities. Intuitively, this situation occurs because profit risk caused by output uncertainty is lowest at low prices.[9] In four of the five cases [(b) through (e)], the producer takes additional insurance when the realized futures price is high. This action can be explained by the positive correlation between realized futures price and profit risk. Case (a) is the only exception to this rule. The payoff diagram for the combined position of case (a) is inversely V-shaped (hereafter denoted Λ-shaped). Here, the effect of production uncertainty is to hedge

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[9] For simplicity, suppose that $\tilde{F} = \tilde{P}$ and that $\tilde{Y}_0$ is the profit after the producer sells $\bar{Q}$ futures, that is, $\tilde{Y}_0 = \tilde{F}\tilde{Q} + (\bar{F} - \tilde{F})\bar{Q} - C(\tilde{Q})$. The conditional expectation and the conditional variance of $\tilde{Y}_0$ on the realized futures price are $E[\tilde{Y}_0|\tilde{F}] = \bar{F}\bar{Q} - C(\bar{Q})$ and $Var[\tilde{Y}_0|\tilde{F}] = \tilde{F}^2 Var(\tilde{Q})$. Therefore, for each realized futures price, the expected profit is constant but the profit variation increases as futures price increases.

against small price changes and to accept losses when price changes are large. This situation occurs when the producer's subjective estimate of output variance is low, when he or she perceives that the possibility of high prices is low, and/or when risk aversion is very low so that the producer is unconcerned about the possibility of high prices.

Interestingly, there is one possibility for which no options are purchased [(d) in Figure 2]. In this case, however, the number of futures contracts is different from that in the LMH model (i.e., $X^* = \beta\bar{Q} + \Delta X^*$, where $\Delta X^* < 0$ because $\Delta X^* < \Delta Z^* = 0$). This possibility leads to the conclusion that production uncertainty creates hedging decisions that are different from the LMH model, regardless of the functional form of the utility or the expected price distribution.

To summarize, in the absence of any anticipated correlation between the producer's output and prices, the effect of production uncertainty on profit risk is greatest near the mean price or at high prices. The producer will hedge against this additional risk by creating payoff schemes that are loss-making at low prices and profit-generating near the mean or at high prices. The optimal hedging position depends on the producer's utility function and expected output and price distributions.

INTRODUCING DEPENDENCE BETWEEN PRICES AND OUTPUT

Consider a circumstance for which local production changes are expected to be correlated with price changes. Following Losq, the aggregate demand ($\widetilde{Q}^d$) and random individual output ($\widetilde{Q}$) faced by the individual producer are

$$\tilde{Q}^d = D(\tilde{P})$$

$$\tilde{Q} = K(\tilde{Q}^s; \tilde{\kappa})$$

(13)

where $\widetilde{Q}^s$ is the aggregate supply and $\widetilde{\kappa}$ represents the component of firm-specific production uncertainty, which does not influence aggregate supply and price. At equilibrium, $\widetilde{Q}^s = \widetilde{Q}^d$ so that the producer's random output is

$$\tilde{Q} = K[D(\tilde{P}); \tilde{\kappa}]$$

(14)

The first derivative of $\widetilde{Q}$ with respect to $\widetilde{P}$ is

$$\frac{d\tilde{Q}}{d\tilde{P}} = \frac{\partial K}{\partial D}\frac{\partial D}{\partial \tilde{P}}$$

Multiplying by $\widetilde{P}/\widetilde{Q}$ on both sides and rearranging, the following relation holds:

$$\tilde{\eta} = \tilde{\eta}_1 \tilde{\eta}_2$$

(15)

where $\tilde{\eta} = \partial \ln(\widetilde{Q})/\partial \ln(\widetilde{P})$, $\tilde{\eta}_1 = \partial \ln(K)/\partial \ln(\widetilde{Q}^s)$, and $\tilde{\eta}_2 = \partial \ln(D)/\partial \ln(\widetilde{P})$.

The elasticity coefficient ($\tilde{\eta}$) is the product of the elasticity of local production with

respect to aggregate supply $(\tilde{\eta}_1)$[10] and the elasticity of the aggregate demand with respect to

price $(\tilde{\eta}_2)$. Assume that aggregate demand has a negative correlation with price (i.e., $\tilde{\eta}_2 < 0$),

that local output has a positive correlation with aggregate supply (i.e., $\tilde{\eta}_1 > 0$), that η is

constant,[11] and that $-1 < \eta < 0$.[12]

From (14), $\tilde{Q}$ is a function of $\tilde{P}$ (in turn, $\tilde{P}$ is a function of $\tilde{F}$ and $\tilde{\epsilon}$) and $\tilde{\kappa}$, that is, $\tilde{Q}$

$= K[D(\tilde{P}), \tilde{\kappa}] = K[D(\tilde{F}, \tilde{\epsilon}), \tilde{\kappa}] = K^*(\tilde{F}, \tilde{\epsilon}, \tilde{\kappa})$. Here, $\tilde{F}, \tilde{\epsilon}$, and $\tilde{\kappa}$ are independent of

each other, and thus the joint density function of $\tilde{F}, \tilde{\epsilon}$, and $\tilde{\kappa}$ is the product of each density

function. This information is useful in analyzing the first-order conditions. Now, we consider

the first-order condition (5.1) in the case where production and price are correlated. This can be

expressed as

$$E[u'(\tilde{Y})(\tilde{F} - \bar{F})] = \int_{\tilde{F}} (\tilde{F} - \bar{F}) \{ \int_{\tilde{\epsilon}} \int_{\tilde{\kappa}} u'(\tilde{Y}) h_1(\tilde{\kappa}) h_2(\tilde{\epsilon}) d\tilde{\kappa} d\tilde{\epsilon} \} h_3(\tilde{F}) d\tilde{F}$$

$$= E[(\tilde{F} - \bar{F}) \{ E[u'(\tilde{Y}) | \tilde{F}] \}] = E[g(\tilde{F})(\tilde{F} - \bar{F})] = 0$$

(16.1)

where $h_1(\tilde{\kappa})$, $h_2(\tilde{\epsilon})$, and $h_3(\tilde{F})$ are density functions of $\tilde{\kappa}, \tilde{\epsilon}$, and $\tilde{F}$, respectively.

Similarly, (5.2) can be rewritten as

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[10] Losq calls $\tilde{\eta}_1$ a pseudo-elasticity, which measures the degree of covariability between local output and aggregate supply.

[11] The producer might believe that farm yields are positively correlated with regional yields and that increases in regional yields can cause national price decreases.

[12] The term η can be less than -1, but we will assume that η is inelastic. In many agricultural commodities, the demand elasticity (η_2) is inelastic and the elasticity of local output with respect to aggregate supply (η_1) is also inelastic. Therefore, it is reasonable to assume that $-1 < \eta \, (=\eta_1 \eta_2) < 0$.

$$E[u'(\tilde{Y})\{(\tilde{F} - \bar{F})L + \tilde{R}\}] = E[g(\tilde{F})\{(\tilde{F} - \bar{F})L + \tilde{R}\}] = 0 \qquad (16.2)$$

Under nonincreasing absolute risk aversion and $-1 < \eta < 0$, $g(\tilde{F})$ is continuous, strictly convex, and differentiable over the intervals $[0, \bar{F}]$ and $[\bar{F}, \infty)$.[13] Applying the mean value theorem to $g(\tilde{F})$ yields

$$\frac{g(\tilde{F}) - g(\bar{F})}{\tilde{F} - \bar{F}} = \frac{dg(\tilde{f})}{d\tilde{f}} = E[u''(\tilde{y})\{\beta(1 + \eta)\tilde{q} - X + LZ\}|\tilde{F}]$$

where $\tilde{q} = Q(\tilde{f})$ and $\tilde{y}$ is the profit of an individual firm when the futures price is $\tilde{f}$. Thus, $g(\tilde{F})$ is

$$g(\tilde{F}) = g(\bar{F}) + (\tilde{F} - \bar{F})E[u''(\tilde{y})\{\beta(1 + \eta)\tilde{q} - X + LZ\}|\tilde{F}] \qquad (17)$$

Substituting (17) into (16.1) and (16.2) and rearranging in a similar manner to that for obtaining (7.1) and (7.2)[14] yields

$$\mathcal{L}_{FF}X^* - \mathcal{L}_{FF1}Z^* = \beta(1 + \eta)d \qquad (18.1)$$

$$- \mathcal{L}_{FF1}X^* + \mathcal{L}_{FF1}Z^* = - \beta(1 + \eta)e + c \qquad (18.2)$$

where $c = - \bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y})|\bar{F}]\}$, $d = E[u''(\tilde{y})\tilde{q}(\tilde{F} - \bar{F})^2]$, $e = E_1[u''(\tilde{y})\tilde{q}(\tilde{F} - \bar{F})^2]$, $\mathcal{L}_{FF} < \mathcal{L}_{FF1} < 0$, and $d < e < 0$.

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[13] This is true because

$$\frac{\partial^2 g(\tilde{F})}{\partial \tilde{F}^2} = E[u'''(\tilde{Y})\{\beta\tilde{Q}(1 + \eta) - X + LZ\}^2|\tilde{F}] + E[u''(\tilde{Y})\beta(1 + \eta)\frac{\partial \tilde{Q}}{\partial \tilde{F}}|\tilde{F}] > 0.$$

[14] The derivations of (18.1) and (18.2) are presented in Appendix D.

The procedure to find the sign of c is similar to that used previously. The term c can be rewritten as follows:

$$c = - \bar{R}E[u''(\tilde{y})(\tilde{F} - \bar{F})\{\beta\tilde{q}(1 + \eta) - X^* + LZ^*\}]$$

$$= - \bar{R}Cov[\tilde{F}, u''(\tilde{y})\{\tilde{q}(1 + \eta) - X^* + LZ^*\}]$$

The covariance term has the same sign as $\partial[u''(\tilde{y})\{\beta\tilde{q}(1 + \eta) - X^* + LZ^*\}]/\partial\tilde{F} =$

$u'''(\tilde{y})\{\beta\tilde{q}(1 + \eta) - X^* + LZ^*\}^2 + u''(\tilde{y})\beta(1 + \eta)(\partial\tilde{q}/\partial\tilde{f})(\partial\tilde{f}/\partial\tilde{F})$, which is positive under

$u''' > 0$ and $-1 < \eta < 0$. Therefore, c is negative.

The optimal futures and options amounts under the dependence assumption are obtained

by solving (18.1) and (18.2) simultaneously. They are

$$X^* = \frac{\{\beta(1 + \eta)(d - e) + c\}\mathcal{L}_{FFI}}{\Delta} \tag{19.1}$$

$$Z^* = \frac{\beta(1 + \eta)(\mathcal{L}_{FFI}d - \mathcal{L}_{FF}e) + \mathcal{L}_{FF}c}{\Delta} \tag{19.2}$$

One result is that, under $u''' > 0$ and $-1 < \eta < 0$, the producer always sells futures

because $d < e < 0$, $c < 0$, and $\mathcal{L}_{FF1} < 0$. Options are almost always required for hedging.

To understand the intuition here, assume that $\tilde{P} = \tilde{F}$ and that individual output has a one-to-one

relationship with price. Because $\partial(\tilde{P}\tilde{Q})/\partial\tilde{P} = \tilde{Q}(1 + \eta) > 0$ and $\partial^2(\tilde{P}\tilde{Q})/\partial\tilde{P}^2 = (1 + \eta)$

$(\partial\tilde{Q}/\partial\tilde{P}) < 0$ if $-1 < \eta < 0$, the unhedged random revenue $(\tilde{P}\tilde{Q})$ is concave in the realized

price and its slope is always positive. In this case, options can be used to hedge against the

nonlinearity of revenue.

NUMERICAL SIMULATIONS

The results obtained from the previous sections are supported by using numerical simulation in this section. This section also analyzes the effect of the degree and nature of risk aversion, the size of the elasticity coefficient (η), and the source of production uncertainty on hedging behavior. First, the method used to find the optimal futures and options amounts is explained. Then, the optimal futures and options positions are calculated under various scenarios. In all cases, we assume that $\tilde{P} = \tilde{F}$ (no basis risk) and that the producer has constant absolute risk aversion (CARA), that is, $u(\tilde{Y}) = m - n \exp[-A\tilde{Y}]$ where A is a constant absolute risk-aversion coefficient and m and n are coefficients. We also assume that $C(\bar{Q}) = 0$ and hence discuss revenue rather than profit.

One motivation for this section is to show that, if one has specific information about risk aversion and subjective expectations about the futures price and output distributions and the correlation between individual yields and national price (if any), one can solve for the optimal futures and options positions for individual producers. The method we use to find these optimal positions is straightforward and lends itself to real-world application. To show that this is possible, we chose data that is relevant to a typical Iowa corn producer. Because several of the variables used in the simulation are not known with certainty, we also perform a sensitivity analysis to show how the optimal positions respond to changes in the data.

Data and Method

The mean and variance of the output for typical Iowa corn producers were calculated from Iowa Farm Costs and Returns (for the years 1970-89). The coefficient of variation of corn

production for the average Iowa farmer was 0.158113.[15] Average 1989 corn production of 20,000 bushels was used to represent mean production and thus the variance of corn production was assumed to be 1 x 10^7.[16]

Corn is assumed to be planted in the second week of May and harvested in the second week of September. We assume that the September corn futures price in September is the mean price for the year. The deviation of the futures price from the mean is calculated as the difference between the September corn futures prices in May and September. We used data from 1974-89. The coefficient of variation of futures price for these years was 0.173205. The September corn futures price in the second week of September 1989 was $2.92 and the variance of futures price was 0.255792.

The optimization procedure used in this study is as follows. The first step is to establish for X an interval of ± 10,000 around a starting point and to divide this interval into 19 evenly spaced segments so that the number of X considered for calculation is 20. For example, if the starting point of X is 0, the values for X are (−10,000, ..., −2,000, −1,000, 0, 1,000, 2,000, ..., 10,000). The values for Z are obtained by using the same method. In the X-Z plane, there is now a grid of 400 points. The expected utility level at each point in the grid is calculated. The second step is to choose the grid point, for example (X_1, Z_1), where the expected utility function is greatest. If the point is an interior solution, then the first step is repeated within an interval of ± 1,000 around (X_1, Z_1) with segment lengths of 100. If an interior solution is found, it is called (X_2, Z_2). If the point is a corner solution, then the first step within an interval of ± 10,000

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[15] The coefficient of output variation is defined as $\{Var[\tilde{Q}/\bar{Q}]\}^{1/2}$.

[16] When expected production is 20,000 bushels and the coefficient of variation is 0.026165, production variance is calculated as $(0.158113 \times 20,000)^2 = 10,466,000$.

around (X_1, Z_1) is repeated. This procedure is repeated until an interior solution is found. Finally, the first and second steps are repeated within an interval of ± 100 around (X_2, Z_2) with segment lengths of 10.

Strictly speaking, the solution to this procedure may not be exactly at the optimum point. However, the maximum deviation from the true optimum (10) is less than the minimum contract size in futures and options markets.[17] We use a CARA utility function so that the second-order condition is always satisfied (see Appendix B).

Results and Discussion

Assume that a producer has performed the analysis just discussed and believes that price and output are normally distributed as follows:

$$\tilde{F} \sim N(2.92, 0.255792)$$

$$\tilde{Q} \sim N(20{,}000, 1 \times 10^7)$$

We assign values ranging from \$1.30 to \$4.54 for $\tilde{F}$ and from 10,000 bushels to 30,000 bushels for $\tilde{Q}$.

Table 1 represents the producer's optimal hedging behavior in various situations. Rows 1 through 5 are the cases for which price and output are independent and rows 6 through 12 assume dependence. The last four columns in Table 1 indicate the slope of the total hedging position and

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[17] A futures contract for corn is 1,000 bushels on the MidAmerica Commodity Exchange and 5,000 bushels on the Chicago Board of Trade.

Table 1. Optimal hedge for a corn producer who expects to harvest 20,000 bushels

	$\tilde{Q}$ & $\tilde{P}$	A = coefficient of absolute risk aversion	η	X^*	Z^*	slope (X^*, Z^*)		slope $(\Delta X^*, \Delta Z^*)$[d]	
						$\tilde{F} \leq \bar{F}$	$\bar{F} \leq \tilde{F}$	$\tilde{F} \leq \bar{F}$	$\bar{F} \leq \tilde{F}$
1	Ind[c]	0.00015		15,720	-330			3,950	4,280
2	Ind	0.00025		13,320	-1,010			5,670	6,680
3	Ind	0.00035		12,010	-950			7,040	7,990
4	Ind	0.00045		11,320	-740			7,940	8,680
5	Ind	0.00015 if Y ≥ 30,000 0.00045 if 25,000 ≤ Y < 30,000 0.00090 if Y < 25,000		12,200	-4,900			2,900	7,800
6[a]	Dep[c]	0.00015	-0.1	17,830	-340	-18,170	-17,830		
7[a]	Dep	0.00015	-0.3	13,370	-1,260	-14,630	-13,370		
8[a]	Dep	0.00015	-0.5	9,200	-1,590	-10,790	-9,200		
9[a]	Dep	0.00015	-0.7	5,370	-1,280	-6,650	-5,370		
10[a]	Dep	0.00015	-0.9	1,830	-340	-2,170	-1,830		
11[b]	Dep	0.00015	-0.5	8,780	-860	-9,640	-8,780		
12[b]	Dep	0.00015 if Y ≥ 45,000 0.00045 if 40,000 ≤ Y < 45,000 0.00090 if Y < 40,000	-0.5	7,040	-3,200	-10,240	-7,040		

[a] Only firm-specific production uncertainty is considered.

[b] Both firm-specific and marketwide production uncertainty are considered.

[c] Ind and Dep indicate the independent case and the dependent case.

[d] Int the dependence case, the additional positions are meaningless because revenue depends on η.

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the additional hedging position over the intervals [1.3, F̄] and [F̄, 4.84].[18]

Changing the absolute risk-aversion coefficient (rows 1 through 4).

Risk-aversion measures range from 0.00015 to 0.00045 (rows 1 through 4), where the latter represents the most risk-averse case.[19] Results for rows 1 through 4 show that $X^* > 0$ and $\Delta Z^* > \Delta X^*$, and that the additional hedging position is positively sloped in the realized price.[20] Production uncertainty causes more revenue uncertainty at higher prices, as explained earlier. Therefore, the risk-averse producer hedges more against these higher prices than does the less risk-averse producer.

Changing the elasticity coefficient (η) and the nature of production uncertainty (rows 6 through 11).

If there is no-firm specific production uncertainty, additional hedging needs will depend on the producer's perception of the elasticity coefficient. To see why this is true, consider the extreme case for which the elasticity coefficient is −1. Any additional production will reduce prices by an amount that maintains revenue, so no additional hedging would be needed.

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[18] The slope is defined as $\partial[(\bar{F} - \tilde{F})X^* + \{\bar{R} - (\bar{F} - \tilde{F})L\}Z^*]/\partial\tilde{F} = -X^* + LZ^*$. However, we can redefine it as $\partial[(5000\times\bar{F} - 5000\times\tilde{F})(X^*/5000) + \{5000\times\bar{R} - (5000\times\bar{F} - 5000\times\tilde{F})L\}(Z^*/5000)]/\partial(5000\times\tilde{F}) = -X^*/5000 + LZ^*/5000$ if the contract is 5,000 bushels. In this case, for example, the slopes of row 1 are 0.3950 in $\tilde{F} \le \tilde{F}$ and 0.4280 in $\tilde{F} \ge \bar{F}$.

[19] Hanson and Ladd use values ranging from 0.00005 to 0.00045. King and Robinson suggest that the absolute risk-aversion coefficient should be concentrated in the range from −0.0001 to 0.001.

[20] In a simulation not reported in Table 1, we used (i) A = 0.00001 and (ii) A = 0.00015 and $\text{Var}(\tilde{Q}) = 1 \times 10^6$. Here, the additional hedging positions are (7,520, 15,640) and (580, 1,250), respectively, which lead to a Λ-shaped curve.

Suppose that the relation between individual output and price is as follows: $\widetilde{Q} = \gamma \widetilde{P}^{\eta}$, where η is the elasticity coefficient defined in (15) and γ is a constant coefficient. Assume that expected production remains at 20,000 bushels so that $E[\widetilde{Q}] = \gamma E[\widetilde{P}^{\eta}] = 20,000$. Then, if we know the price distribution (assumed to be normally distributed with mean 0 and variance 0.255792) and assume a value for η, we can solve for γ.

Rows 6 through 10 show how the optimal position responds to the elasticity coefficient. All these positions are kinked in futures prices because the producer attempts to offset unhedged revenue patterns that are concave in the realized futures price. The put options amount purchased by the producer is at a maximum when $\eta = -0.5$ (row 8).[21]

Now, suppose that the firm-specific production uncertainty $(\widetilde{\kappa})$ exhibits multiplicative risk as follows:

$$\widetilde{Q} = \gamma \widetilde{P}^{\eta}(1 + \widetilde{\kappa})$$

where $\widetilde{\kappa}$ is assumed to be independent of $\widetilde{P}$ and normally distributed with mean 0. $\text{Var}(\widetilde{\kappa})$ can be obtained from $\text{Var}(\widetilde{Q}) = 1 \times 10^7$ for consistency. Suppose that $\eta = -0.5$ and $A = 0.00015$ (row 11). Thus, $\gamma = 33775.513$ and $\text{Var}(\kappa) = 0.016446957$.[22] The optimal futures and put options amounts are $(8,780, - 860)$ in this case and $(9,200, -1,590)$ when firm-specific production uncertainty was not considered. To hedge this firm-specific production uncertainty, the producer

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[21] $\partial^2(\widetilde{P}\,\widetilde{Q})/\partial \widetilde{P}^{\,2} = \eta(1 + \eta)(\widetilde{P}/\widetilde{Q})$. This equation reflects the curvature of the realized revenue curve with repsect to price. The second derivative with respect to price has a minimum value at $\eta = -0.5$.

[22] The terms γ and $\text{Var}(\widetilde{\kappa})$ can be obtained from $\gamma = 20,000/E[\widetilde{P}^{\eta}]$ and $\text{Var}(\widetilde{\kappa}) = [1 \times 10^7/\gamma^2 + \{E[\widetilde{P}^{\eta}]\}^2 - E[\widetilde{P}^{\,2\eta}]]/E[\widetilde{P}^{\,2\eta}]$.

sells 420 fewer bushels in the futures market and buys 730 fewer bushels in the put options market. Therefore, the additional position attributable to firm-specific production uncertainty is positively sloped in prices.

Revenue-dependent risk aversion (rows 7 and 12).

It is possible to solve the simulation model with absolute risk-aversion coefficients that change with revenue. There may be producers who are less risk averse at high revenue levels and/or very risk averse at low levels. Rows 5 and 12 show the situation for which the degree of risk aversion is negatively correlated with revenue.[23] The results show that more put options are purchased in this situation. The intuition here is that the producer becomes more concerned about lower revenues and therefore purchases more put options than in cases for which we assume constant risk aversion.

The minimum number of bushels in a put option contract is 5,000; therefore, only producers represented by rows 5 and 12 would find it optimal to purchase a put. These numbers are, however, relevant for producers who expect to sell only 20,000 bushels. If expected production was higher, the volume of options required would also be higher. Also, producers in other states may be exposed to more weather-related yield risk than are those in Iowa. More

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[23] We divided the revenue range into three regions: for some value Y_f, region 1 is $Y_f \le \tilde{Y}$, region 2 is $Y_f - 5,000 \le \tilde{Y} < Y_f$, and region 3 is $\tilde{Y} < Y_f - 5,000$. We assign the absolute risk-aversion coefficients of 0.00015 for region 1, 0.00045 for region 2, and 0.00090 for region 3. The CARA utility function is $u_i(\tilde{Y}) = m_i - n_i \exp[-A_i(\tilde{Y} - Y_f)]$ for each region. Continuity and differentiability of the utility function at Y_f indicate that $u_1(0) = u_2(0)$ and $u'_1(0) = u'_2(0)$. Similarly, continuity and differentiability at $(Y_f - 5,000)$ require that $u_2(-5,000) = u_3(-5,000)$ and $u'_2(-5,000) = u'_3(-5,000)$. If $m_1 = 2$ and $n_1 = 3$ are chosen, then $m_2 = 0$, $n_2 = 3$, $m_3 = 1458742.7$, and $n_3 = 180.034626$. This utility function is continuous and differentiable, even though it is a combination of three different utility functions.

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importantly, individual Iowa producers may face significantly greater yield variation than the average for the state as a whole.

In general, the results show that production uncertainty reduces the usefulness and use of futures contracts and increases the usefulness and use of purchased put options.

CONCLUSIONS

Options are a relatively new and popular investment tool for farmers. In the absence of production risk, these assets have no role as hedgng instruments because, with linear price assumption, futures dominate options as a way to offset price risk. When production uncertainty is introduced, however, options have a role to play. For example, if yields are lower than expected and if the producer has sold the expected yield on the futures market, the producer is exposed to revenue risk that can be partially offset with options because he or she will have sold more on the futures market than is available to sell on cash markets. Options are useful whether or not producers believe that their individual yields are correlated with market prices. In addition, the usefulness of options as hedging tools increases with firm-specific production uncertainty and for producers who are more risk averse at lower revenues.

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APPENDIX A: APPLICATION OF THE MEAN VALUE THEOREM

The mean value theorem states:

Let g be a continuous function on [a, b] and have a derivative at all x in [a, b] except at perhaps x = a and x = b. Then there is at least one argument X such that a < X < b and

$$g'(X) = \frac{g(b) - g(a)}{b - a}$$

The conditions for using the mean value theorem are continuity and differentiability. With the existence of options, price distribution is truncated at the strike price and thus $g(\tilde{F})$ is not differentiable at $\overline{F}$. That is, because $L = 1$ when $\tilde{F}$ approaches $\overline{F}$ from the left side and $L = 0$ when $\tilde{F}$ approaches $\overline{F}$ from the right side, the slopes of $g(\tilde{F})$ with respect to $\tilde{F}$ at $\overline{F}^+$ and $\overline{F}^-$ are

$$\lim_{\tilde{F} \to \overline{F}^+} \frac{\partial g(\tilde{F})}{\partial \tilde{F}} = E[u''(\tilde{Y})(\beta\tilde{Q} - X)|\tilde{F} \to \overline{F}^+]$$

and

$$\lim_{\tilde{F} \to \overline{F}^-} \frac{\partial g(\tilde{F})}{\partial \tilde{F}} = E[u''(\tilde{Y})(\beta\tilde{Q} - X + Z)|\tilde{F} \to \overline{F}^-]$$

thus

$$if\ Z \neq 0,\ \lim_{\tilde{F} \to \overline{F}^+} \frac{\partial g(\tilde{F})}{\partial \tilde{F}} \neq \lim_{\tilde{F} \to \overline{F}^-} \frac{\partial g(\tilde{F})}{\partial \tilde{F}}$$

When $\tilde{F}$ approaches $\overline{F}$ from the left side, the slope of $g(\tilde{F})$ is different from the slope of $g(\tilde{F})$ when $\tilde{F}$ approaches $\overline{F}$ from the right side. Therefore, $g(\tilde{F})$ is not differentiable at $\overline{F}$ (although it is continuous). However, $g(\tilde{F})$ is differentiable over the interval $[0, \overline{F}]$ and $[\overline{F}, \infty)$, and thus the mean value theorem can be applied to $g(\tilde{F})$ in $[0, \overline{F}]$ and $[\overline{F}, \infty)$.
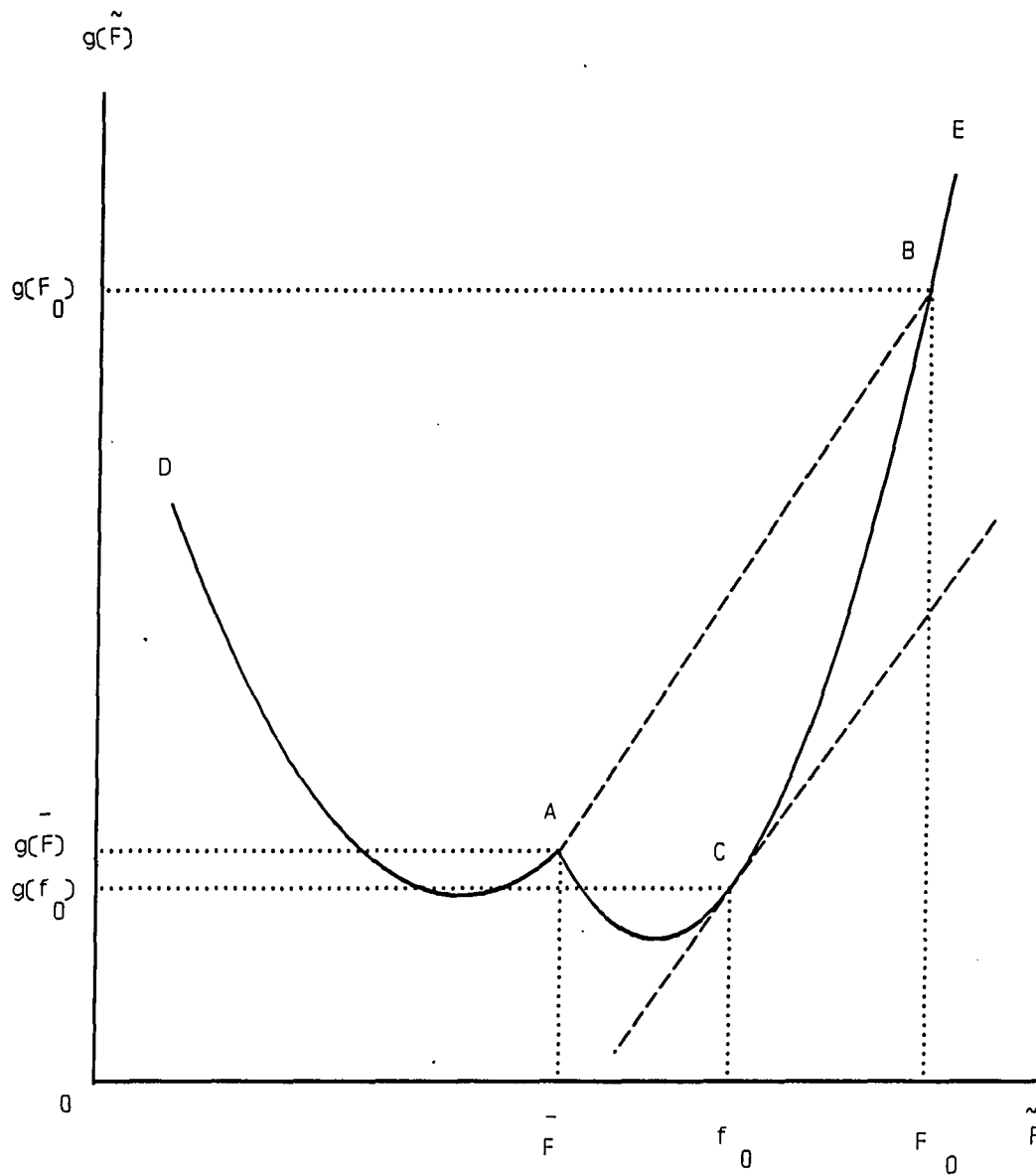
Figure A. A schematic representation of how the expected marginal utility conditioned on realized futures price reponds to the futures price using the mean value theorem

Before proceeding, note that the function $g(\widetilde{F})$ is strictly convex in $\widetilde{F}$ over the interval

$[0, \overline{F}]$ or $[\overline{F}, \infty)$, which can be shown by differentiating $g(\widetilde{F})$ with respect to $\widetilde{F}$ as follows:

$$\frac{\partial^2 g(\widetilde{F})}{\partial \widetilde{F}^2} = E[u'''(\widetilde{Y})(\beta \bar{Q} - X + LZ)^2 | \widetilde{F}]$$

To characterize the sign of $\partial^2 g(\widetilde{F})/\partial \widetilde{F}^2$, we need to sign $u'''(\widetilde{Y})$. First, consider that absolute

risk aversion is given by $A = -u''(\widetilde{Y})/u'(\widetilde{Y})$. Then, nonincreasing absolute risk aversion

means that $\partial A/\partial \widetilde{Y} = -[u'''(\widetilde{Y})/u'(\widetilde{Y})] + [u''(\widetilde{Y})/u'(\widetilde{Y})]^2 \leq 0$. This implies that u''' must be

positive. Therefore, under nonincreasing absolute risk aversion, the second derivative of $g(\widetilde{F})$

with respect to $\widetilde{F}$ is positive because both terms within the expectation are positive.

Consequently, under nonincreasing

absolute risk aversion, $g(\widetilde{F})$ is strictly convex and differentiable in $\widetilde{F}$ over the interval $[0, \overline{F}]$ or

$[\overline{F}, \infty)$.

Figure A shows how the mean value theorem can be applied to $g(\widetilde{F})$ over the interval

$[\overline{F}, \infty)$. Suppose that curve DACBE represents $g(\widetilde{F})$. There is a futures price f_0 such that $g'(f_0)$

will be the same as the slope of line AB. Here, for F_0 in $[\overline{F}, \infty)$, f_0 is unique given the strict

convexity of $g(\widetilde{F})$ in $\overline{F}, \infty)$. Equivalently, the slope of the line connecting $(\overline{F}, g(\overline{F}))$ and

$(\widetilde{F}, g(\widetilde{F}))$ for any $\widetilde{F}$ in $\widetilde{F} \geq \overline{F}$ is the same as $g'(\tilde{f})$, that is,

$$\frac{g(\widetilde{F}) - g(\overline{F})}{\widetilde{F} - \overline{F}} = \frac{\partial g(\tilde{f})}{\partial f} = E[u''(\tilde{y})(\beta \bar{Q} - X + LZ) | \tilde{F}] \qquad (A.1)$$

where $\tilde{y} = (r + \beta \tilde{f} + \tilde{\epsilon})\bar{Q} + (\overline{F} - \tilde{f})X + \{\bar{R} - (\overline{F} - \tilde{f})L\}Z)$ and $\tilde{f}$ is a monotonically

increasing function of $\widetilde{F}$ because $g(\widetilde{F})$ is strictly convex in $[0, \overline{F}]$ and $[\overline{F}, \infty)$. The left-hand side

in (A.1) represents the slope of line AB, and the right-hand side represents the slope of $g(\widetilde{F})$ at

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$\widetilde{F} = f_0$. This analysis can be conducted for all $\widetilde{F}$ in $[\overline{F}, \infty)$ by connecting point A and any

point on curve $g(\widetilde{F})$. Applying the mean value theorem to $g(\widetilde{F})$ over the interval $[0, \overline{F}]$ is similar

to the explanation provided here.

From A.1, $g(\widetilde{F})$ is

$$g(\tilde{F}) = g(\bar{F}) + (\tilde{F} - \bar{F})g'(\hat{f}) = g(\bar{F}) + (\tilde{F} - \bar{F})E[u''(\tilde{y})(\beta\tilde{Q} - X + L\tilde{Z})|\tilde{F}]$$

APPENDIX B: THE SECOND-ORDER CONDITION
UNDER A CARA UTILITY UNCTION

Given the existence of futures and options markets, the second-order condition is

$$SOC = E[u''(\tilde{Y})(\bar{F} - \tilde{F})^2]E[u''(\tilde{Y})\{\bar{R} - (\bar{F} - \tilde{F})L\}^2]$$

$$- \{E[u''(\tilde{Y})(\bar{F} - \tilde{F})\{\bar{R} - (\bar{F} - \tilde{F})L\}]\}^2 \tag{B.1}$$

where

$$E[u''(\tilde{Y})\{\bar{R} - (\bar{F} - \tilde{F})L\}^2]$$

$$= \bar{R}^2 E[u''(\tilde{Y})] - 2\bar{R}\alpha E_1[u''(\tilde{Y})(\tilde{F} - \bar{F})] + \alpha E_1[u''(\tilde{Y})(\tilde{F} - \bar{F})^2]$$

$$E[u''(\tilde{Y})(\bar{F} - \tilde{F})\{\bar{R} - (\bar{F} - \tilde{F})L\}]$$

$$= -\bar{R}E[u''(\tilde{Y})(\tilde{F} - \bar{F})] - \alpha E_1[u''(\tilde{Y})(\tilde{F} - \bar{F})^2]$$

Under a CARA utility function, that is, $E[u''(\tilde{Y})(\tilde{F} - \bar{F})] = 0$, the second order condition in (B.1) can be rearranged as follows:

$$SOC = E[u''(\tilde{Y})(\bar{F} - \tilde{F})^2]\{\bar{R}^2 E[u''(\tilde{Y})] - 2\bar{R}\alpha E_1[u''(\tilde{Y})(\tilde{F} - \bar{F})]\}$$

$$+ \alpha E_1[u''(\tilde{Y})(\tilde{F} - \bar{F})^2]\{E[u''(\tilde{Y})(\tilde{F} - \bar{F})^2] - \alpha E_1[u''(\tilde{Y})(\tilde{F} - \bar{F})^2]\}$$

which is always positive because $E[u''(\tilde{Y})(\tilde{F} - \bar{F})^2] < \alpha E_1[u''(\tilde{Y})(\tilde{F} - \bar{F})^2] < 0$ and $E_1[u''(\tilde{Y})(\tilde{F} - \bar{F})] > 0$. Consequently, using a CARA utility function, the second-order condition is always satisfied.

APPENDIX C: DERIVATION OF EQUATIONS (7.1) AND (7.2)

The first-order conditions are

$$E[g(\tilde{F})(\tilde{F} - \bar{F})] = 0 \qquad \text{(C.1)}$$

$$E[g(\tilde{F})\{(\tilde{F} - \bar{F})L + \bar{R}\}] = 0 \qquad \text{(C.2)}$$

where $L = 1$ if $\tilde{F} \le \bar{F}$ and $L = 0$ if $\tilde{F} \ge \bar{F}$, and $g(\tilde{F}) \equiv E[u'(\tilde{Y})|\tilde{F}]$. In Appendix A, it was shown that application of the mean value theorem to $g(\tilde{F})$ gave

$$g(\tilde{F}) = g(\bar{F}) + (\tilde{F} - \bar{F})E[u''(\tilde{y})(\beta\tilde{Q} - X + LZ)|\tilde{F}] \qquad \text{(C.3)}$$

where $\tilde{y}$ is profit associated with a futures price of $\tilde{f}$. Substituting (C.3) into (C.1) gives

$$E[(\tilde{F} - \bar{F})g(\bar{F})] + E[(\tilde{F} - \bar{F})^2\{E[u''(\tilde{y})(\beta\tilde{Q} - X + LZ)|\tilde{F}]\}] = 0 \qquad \text{(C.4)}$$

Because $g(\bar{F})$ is a fixed number and $E[E\{\bullet|\tilde{F}\}] = E[\bullet]$, (C.4) can be rewritten as

$$E[(\tilde{F} - \bar{F})]g(\bar{F}) + E[u''(\tilde{y})(\tilde{F} - \bar{F})^2(\beta\tilde{Q} - X + LZ)] = 0$$

Because $E[\tilde{F} - \bar{F}] = 0$, (C.4) can be rewritten as follows:

$$E[u''(\tilde{y})(\tilde{F} - \bar{F})^2(\beta\tilde{Q} - X + LZ)] = 0 \qquad \text{(C.5)}$$

Equation (C.2) can be rewritten in a similar manner:

$$\alpha E_1[g(\tilde{F})(\tilde{F} - \bar{F})] + \bar{R}E[u'(\tilde{Y})] = 0 \qquad \text{(C.6)}$$

where $\alpha = \text{Prob}[\tilde{F} \le \bar{F}]$ and subscript 1 represents the conditional expectation on $\tilde{F} \le \bar{F}$, that is,

$E_1[\bullet] = E[\bullet | \tilde{F} \leq \bar{F}]$.

The first term (FT) in the left-hand side of (C.6) can be written, by substituting for $g(\tilde{F})$ from (C.3), as

$$FT = \alpha E_1[(\tilde{F} - \bar{F})\{g(\bar{F}) + (\tilde{F} - \bar{F})E[u''(\tilde{y})(\beta \tilde{Q} - X + LZ) | \tilde{F}]\}]$$

Factoring out terms within $\{\bullet\}$ and using $E[E\{\bullet | \tilde{F}\}] = E[\bullet]$ and $L = 1$ when $\tilde{F} \leq \bar{F}$, it follows that

$$FT = \alpha g(\bar{F})E_1[\tilde{F} - \bar{F}] + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2(\beta \tilde{Q} - X + Z)]$$

Now using $\bar{R} = E[(\tilde{F} - \tilde{F})L] = -\alpha E_1[\tilde{F} - \bar{F}]$ and $g(\bar{F}) = E[u'(\tilde{Y}) | \tilde{F} = \bar{F}]$ yields

$$FT = -\bar{R}E[u'(\tilde{Y}) | \tilde{F} = \bar{F}] + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2(\beta \tilde{Q} - X + Z)]$$

Therefore, (C.6) can be rewritten as

$$-\bar{R}E[u'(\tilde{Y}) | \tilde{F} = \bar{F}] + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2(\beta \tilde{Q} - X + Z)] + \bar{R}E[u'(\tilde{Y})]$$

$$= \bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y}) | \tilde{F} = \bar{F}]\} + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2(\beta \tilde{Q} - X + Z)] = 0$$

(C.7)

Using (C.5) and (C.7), the first-order conditions are:

$$E[u''(\tilde{y})(\tilde{F} - \bar{F})^2\{\beta \tilde{Q} - X + LZ\}] = 0 \qquad (C.8)$$

$$\bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y}) | \tilde{F} = \bar{F}]\} + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2\{\beta \tilde{Q} - X + Z\}] = 0 \quad (C.9)$$

Factoring out terms in $\{\bullet\}$, (C.8) and (C.9) at the optimum can be rewritten as follows:

$$\beta E[u'''(\tilde{y})(\tilde{F} - \bar{F})^2 \tilde{Q}] - X^* E[u'''(\tilde{y})(\tilde{F} - \bar{F})^2]$$

$$+ Z^* \alpha E_1[u'''(\tilde{y})(\tilde{F} - \bar{F})^2] = 0 \tag{C.10}$$

$$\bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y})|\tilde{F} = \bar{F}]\} + \beta \alpha E_1[u'''(\tilde{y})(\tilde{F} - \bar{F})^2 \tilde{Q}]$$

$$- X^* \alpha E_1[u'''(\tilde{y})(\tilde{F} - \bar{F})^2] + Z^* \alpha E_1[u'''(\tilde{y})(\tilde{F} - \bar{F})^2] = 0 \tag{C.11}$$

Consequently, (C.10) and (C.11) can be rearranged as follows:

$$\mathscr{L}_{FF}X^* - \mathscr{L}_{FFI}Z^* = \beta a \tag{C.12}$$

$$- \mathscr{L}_{FFI}X^* + \mathscr{L}_{FFI}Z^* = -\beta b + c \tag{C.13}$$

where $\mathscr{L}_{FF} = E[u''(\tilde{F} - \bar{F})^2]$, $\mathscr{L}_{FFI} = \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2]$, $a = E[u''(\tilde{y})\tilde{Q}(\tilde{F} - \bar{F})^2]$, $b = \alpha E_1[u''(\tilde{y})\tilde{Q}(\tilde{F} - \bar{F})^2]$, and $c = -\bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y})|\tilde{F} = \bar{F}]\}$. This gives a system of equations in the variables X and Z. If $\mathscr{L}_{FF}$, $\mathscr{L}_{FFI}$, a, b, and c are constant, this system could be easily solved for X and Z.

APPENDIX D: DERIVATION OF EQUATIONS (18.1) AND (18.2)

The first-order conditions are

$$E[g(\tilde{F})(\tilde{F} - \bar{F})] = 0 \qquad (D.1)$$

$$E[g(\tilde{F})\{(\tilde{F} - \bar{F})L + \bar{R}\}] = 0 \qquad (D.2)$$

where $L = 1$ if $\tilde{F} \leq \bar{F}$ and $L = 0$ if $\tilde{F} \geq \bar{F}$, and $g(\tilde{F}) = E[u'(\tilde{Y})|\tilde{F}]$. In Appendix A, it was shown that application of the mean value theorem to $g(\tilde{F})$ gave

$$g(\tilde{F}) = g(\bar{F}) + (\tilde{F} - \bar{F})E[u''(\tilde{y})\{\beta(1 + \eta)\tilde{q} - X + LZ\}|\tilde{F}] \qquad (D.3)$$

where $\tilde{q} = q(\tilde{f})$ and $\tilde{y}$ is profit associated with a futures price of $\tilde{f}$. Substituting (D.3) into (D.1) gives

$$E[(\tilde{F} - \bar{F})g(\bar{F})] + E[(\tilde{F} - \bar{F})^2\{E[u''(\tilde{y})(\beta\tilde{q} - X + LZ|\tilde{F}]\}] = 0 \qquad (D.4)$$

Because $g(\bar{F})$ is a fixed number and $E[E\{\bullet|\tilde{F}\}] = E[\bullet]$, (D.4) can be rewritten as

$$E[(\tilde{F} - \bar{F})]g(\bar{F}) + E[u''(\tilde{y})(\tilde{F} - \bar{F})^2\{\beta(1 + \eta)\tilde{q} - X + LZ\}] = 0$$

Because $E[\tilde{F} - \bar{F}] = 0$, (D.4) can be rewritten as follows:

$$E[u''(\tilde{y})(\tilde{F} - \bar{F})^2\{\beta(1 + \eta)\tilde{q} - X + LZ\}] = 0 \qquad (D.5)$$

Equation (D.2) can be rewritten in a similar manner:

$$\alpha E_1[g(\tilde{F})(\tilde{F} - \bar{F})] + \bar{R}E[u'(\tilde{Y})] = 0 \qquad \text{(D.6)}$$

where $\alpha = \text{Prob}[\tilde{F} \le \bar{F}]$ and subscript 1 represents the conditional expectation on $\tilde{F} \le \bar{F}$, that is, $E_1[\bullet] = E[\bullet | \tilde{F} \le \bar{F}]$.

The first term in the left-hand side of (D.6) (FT) can be written, by substituting for $g(\tilde{F})$ from (D.3), as

$$FT = \alpha E_1[(\tilde{F} - \bar{F})\{g(\bar{F}) + (\tilde{F} - \bar{F})E[u''(\tilde{y})\{\beta(1 + \eta)\tilde{q} - X + LZ\}|\tilde{F}]\}]$$

Factoring out terms within $\{\bullet\}$, and using $E[E\{\bullet | \tilde{F}\}] = E[\bullet]$ and $L = 1$ when $\tilde{F} \le \bar{F}$, it follows that

$$FT = \alpha g(\bar{F})E_1[\tilde{F} - \bar{F}] + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2\{\beta(1 + \eta)\tilde{q} - X + Z\}]$$

Now using $\bar{R} = E[(\tilde{F} - \tilde{F})L] = -\alpha E_1[\tilde{F} - \bar{F}]$ and $g(\bar{F}) = E[u'(\tilde{Y})|\tilde{F} = \bar{F}]$ yields

$$FT = -\bar{R}E[u'(\tilde{Y})|\tilde{F} = \bar{F}] + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2\{\beta(1 + \eta)\tilde{q} - X + Z\}]$$

Therefore, (D.6) can be rewritten as

$$\bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y})|\tilde{F} = \bar{F}]\} + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2\{\beta(1 + \eta)\tilde{q} - X + Z\}] = 0 \quad \text{(D.7)}$$

Using (D.5) and (D.7), the first-order conditions are:

$$E[u''(\tilde{y})(\tilde{F} - \bar{F})^2\{\beta(1 + \eta)\tilde{q} - X + LZ\}] = 0 \qquad \text{(D.8)}$$

$$\bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y})|\tilde{F} = \bar{F}]\} + \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2\{\beta(1 + \eta)\tilde{q} - X + Z\}] = 0 \text{ (D.9)}$$

Factoring out terms in {•}, (D.8) and (D.9) at the optimum can be rewritten as follows

$$\beta(1 + \eta)E[u''(\tilde{y})(\tilde{F} - \bar{F})^2\tilde{q}] - X^*E[u''(\tilde{y})(\tilde{F} - \bar{F})^2]$$
$$+ Z^*\alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2] = 0 \tag{D.10}$$

$$\bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y})|\tilde{F} = \bar{F}]\} + \beta\alpha(1 + \eta)E_1[u'''(\tilde{y})(\tilde{F} - \bar{F})^2\tilde{q}]$$
$$- X^*\alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2] + Z^*\alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2] = 0 \tag{D.11}$$

Consequently, (D.10) and (D.11) can be rearranged as follows:

$$\mathscr{L}_{FF}X^* - \mathscr{L}_{FF1}Z^* = \beta(1 + \eta)a \tag{D.12}$$

$$- \mathscr{L}_{FF1}X^* + \mathscr{L}_{FF1}Z^* = - \beta(1 + \eta)b + c \tag{D.13}$$

where $\mathscr{L}_{FF} = E[u''(\tilde{F} - \bar{F})^2]$, $\mathscr{L}_{FF1} = \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})^2]$, $d = E[u''(\tilde{y})\tilde{q}(\tilde{F} - \bar{F})^2]$, $e = \alpha E_1[u''(\tilde{y})\tilde{q}(\tilde{F} - \bar{F})^2]$, and $c = - \bar{R}\{E[u'(\tilde{Y})] - E[u'(\tilde{Y})|\tilde{F} = \bar{F}]\}$. This gives a system of equations in the variables X and Z. If $\mathscr{L}_{FF}$, $\mathscr{L}_{FF1}$, a, b, and c are constant, this system could be easily solved for X and Z.

GENERAL SUMMARY

The methodology used in the first paper has many possible applications. For example, one could determine whether generic or branded advertizing campaigns have been successful. One could also measure the impact of societal changes on demand for commodities or commodities aggregates. Finally, one could assume that consumer preferences are constant and check for structural change before using data for econometric purposes.

The second paper used the mean value theorem to examine expected utility maximization hedging behavior in the presence of both price and output uncertainty when futures and options are available. Although options truncate the price distribution at the strike price, this paper shows that one can use the mean value theorem to obtain the optimal futures and options position. Other institutions such as insurance and some government programs truncate the distribution of price or revenue. The application of the mean value theorem presented in this paper is therefore useful for situations other than that considered here.

The third paper shows that options are a popular tool for farmers. Options can be used as a hedging instrument when production uncertainty is introduced while these assets have no role to play in the absence of production risk. The usefulness of options as hedging tools increases with firm-production uncertainty and for producers who are more risk averse at lower revenues.

ACKNOWLEDGEMENTS